Lectures on Supersymmetry I

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The Standard Model + Gravity

- The Standard Model is based on a chiral gauge theory and provides a good theoretical description of all phenomena observed at high energies (see M. Quiros lectures)
- It includes the electromagnetic, the weak and the strong interactions, which lead to an explanation of all known chemical and nuclear processes
- Although presumably incomplete, combined with general relativity, it provides an extremely successful theory for the description of processes in nature
- Let me summarize then the SM basic properties and raise some open questions, that motivate the introduction of Supersymmetry

Standard Model Particles

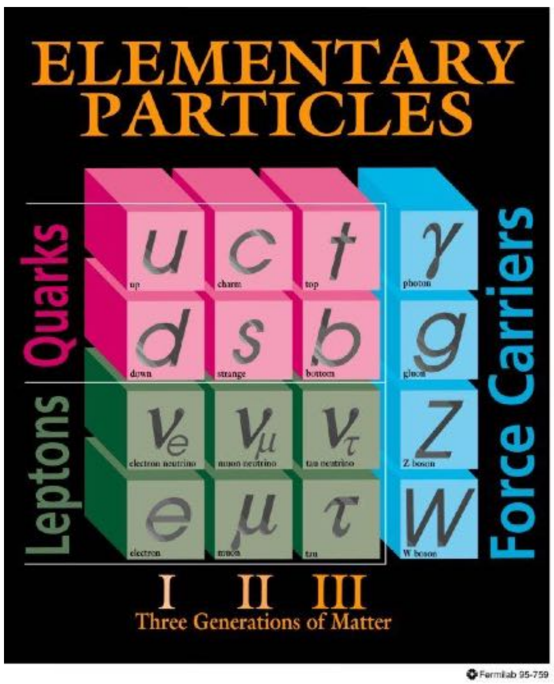
There are 12 fundamental gauge fields:

8 gluons, 3 W_μ's and B_μ

and 3 gauge couplings g_1, g_2, g_3

The matter fields:

3 families of quarks and leptons with same quantum numbers under gauge groups



But very different masses!

m_3/m_2 and $m_2/m_1 \simeq$ a few tens or hundreds

$m_e = 0.5\ 10^{-3}$ GeV, $\frac{m_\mu}{m_e} \simeq 200$, $\frac{m_\tau}{m_\mu} \simeq 20$

Largest hierarchies

$m_t \simeq 175$ GeV $\qquad m_t/m_e \propto 10^5$

neutrino masses smaller than as 10^{-9} GeV!

FERMIONS matter constituents spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	−1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	−1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	−1

Quarks spin = 1/2

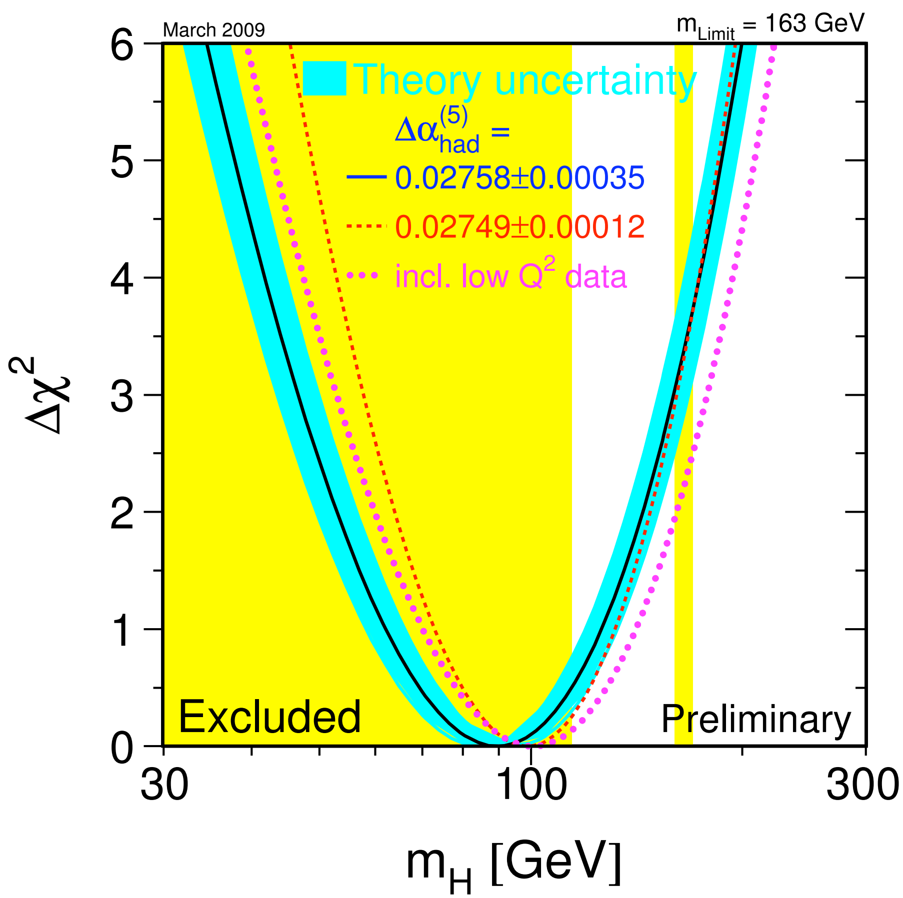
Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.003	2/3
d down	0.006	−1/3
c charm	1.3	2/3
s strange	0.1	−1/3
t top	175	2/3
b bottom	4.3	−1/3

Only left handed fermions transform under the weak SM gauge group $SU(3) \times SU(2)_L \times U(1)_Y$

Fermion and gauge boson masses forbidden by symmetry

SM: Consistent picture of physics at or below the weak scale

Sensitivity to the loop-induced Higgs quantum corrections



	Measurement	Fit	$\lvert O^{meas}-O^{fit}\rvert/\sigma^{meas}$
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02767	
m_Z [GeV]	91.1875 ± 0.0021	91.1874	
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959	
σ_{had}^0 [nb]	41.540 ± 0.037	41.478	
R_l	20.767 ± 0.025	20.742	
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01643	
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480	
R_b	0.21629 ± 0.00066	0.21579	
R_c	0.1721 ± 0.0030	0.1723	
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038	
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742	
A_b	0.923 ± 0.020	0.935	
A_c	0.670 ± 0.027	0.668	
A_l(SLD)	0.1513 ± 0.0021	0.1480	
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314	
m_W [GeV]	80.399 ± 0.025	80.378	
Γ_W [GeV]	2.098 ± 0.048	2.092	
m_t [GeV]	173.1 ± 1.3	173.2	

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The Standard Model is a Chiral Gauge Theory

SM particle	G_{SM}
(S = 1/2)	$SU(3)_c \times SU(2)_L \times U(1)_Y$
$Q = (t, b)_L$	(3,2,1/6)
$L = (\nu, l)_L$	(1,2,-1/2)
$U = \left(t^C\right)_L$	($\bar{3}$,1,-2/3)
$D = \left(b^C\right)_L$	($\bar{3}$,1,1/3)
$E = \left(l^C\right)_L$	(1,1,1)
(S = 1)	
B_μ	(1,1,0)
W_μ	(1,3,0)
g_μ	(8,1,0)

New Physics at the Weak Scale

- The Standard Model (SM) has provided an understanding of all data collected in low and high energy physics experiments (see M. Quiros lectures).
- However, there are reasons to believe that there is new physics at the weak scale. They are related to both particle physics and cosmology:
- Electroweak Symmetry Breaking: Higgs mechanism
- Source of Dark Matter
- Origin of the Matter-Antimatter asymmetry
- There are other open questions in the SM, like the explanation of the fermion mass hierarchies and mixing angles (including the tiny neutrino masses), the solution to the strong CP problem and dark energy. They are most probably only indirectly related to new physics at the weak scale and therefore I will not discuss them in any detail in these lectures.

Beyond the Standard Models

Japanese couple wins World Tango competition

Mon Aug 31, 10:15 am ET

BUENOS AIRES – A Japanese couple outdanced the Argentines and more to win the traditional salon category at the Tango Dance World Championship.

Absence of Scalars

- There are no fundamental scalars seen in nature

- Could there be any reason for their absence at low energies ?

- The gauge symmetries of the SM forbid fermion and gauge boson masses, but a scalar mass $m_\Phi^2 \Phi^\dagger \Phi$ is allowed by the gauge symmetry.

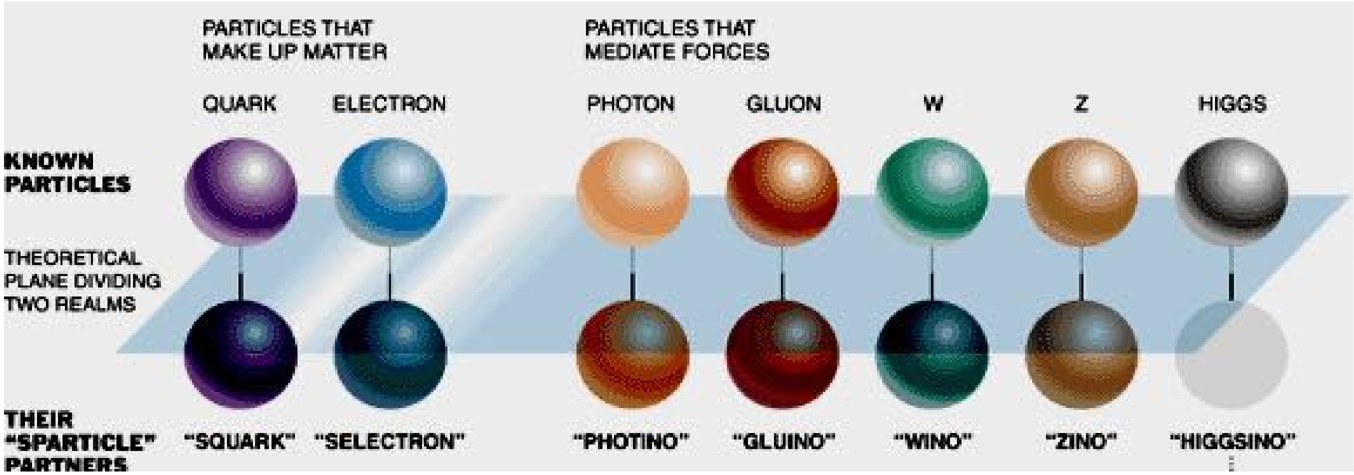
- Since there are no explicit mass parameters, the natural scale for this mass term is of the order of the cutoff of the theory.

- The answer is therefore that scalars are not present because they are naturally heavy.

- But what about the Higgs, then ? Is the cutoff of the SM of the order of the weak scale ? Shouldn't there be many scalars with masses of the same order ? Supersymmetry is a symmetry that demands such a situation, with an effective SM cutoff equal to the SUSY breaking scale.

Supersymmetry

fermions **bosons**



Photino, Zino and Neutral Higgsino: Neutralinos

Charged Wino, charged Higgsino: Charginos

Particles and Sparticles share the same couplings to the Higgs. Two superpartners of the two quarks (one for each chirality) couple strongly to the Higgs with a Yukawa coupling of order one (same as the top-quark Yukawa coupling)

$$\text{Two Higgs doublets necessary} \rightarrow \tan\beta = \frac{v_2}{v_1}$$

Further inspection of the scale hierarchy problem

Spontaneous Symmetry Breakdown

Particle Masses arise through the Higgs mechanism: Spontaneous breakdown of gauge symmetry

$$SU(3)_c \times SU(2)_L \times U(1)_Y \to SU(3)_C \times U(1)_{\rm em} \tag{1}$$

A scalar field, charged under the gauge group, acquires v.e.v.

$$V(H) = m_H^2 H^\dagger H + \frac{\lambda}{2}\left(H^\dagger H\right)^2 \tag{2}$$

Therefore,

$$\left\langle H^\dagger H \right\rangle = -\frac{m_H^2}{\lambda} \tag{3}$$

the v.e.v. of the Higgs field is fixed by the value of the negative mass parameter.

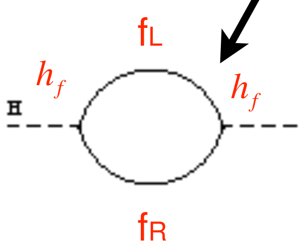
Problem: The mass parameter is unstable under quantum corrections.

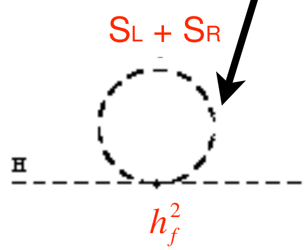
Higgs Mass Parameter Corrections

Quadratic Divergent contributions:

One loop corrections to the Higgs mass parameter cancel if the couplings of scalars and fermions are equal to each other

$$\delta m_H^2 = \frac{N_C h_f^2}{16\pi^2}\left[-2\Lambda^2 + 3m_f^2\log\left(\frac{\Lambda^2}{m_f^2}\right) + 2\Lambda^2 - 2m_s^2\log\left(\frac{\Lambda^2}{m_s^2}\right)\right]$$





(If the masses proceed from the v.e.v. of H, there is another diagram that ensures also the cancellation of the log term. Observe that the fermion and scalar masses are the same in this case, equal to h_f v.)

Supersymmetry is a symmetry that ensures the equality of these couplings

Why Supersymmetry ?

- Helps to stabilize the weak scale—Planck scale hierarchy: $\delta\, m_H^2 \approx (-1)^{2S_i} \dfrac{n_i\, g_i^2}{16\pi^2} \Lambda^2$
- Supersymmetry algebra contains the generator of space-time translations.
  Possible ingredient of theory of quantum gravity.
- Minimal supersymmetric extension of the SM :
  Leads to Unification of gauge couplings.
- Starting from positive masses at high energies, electroweak symmetry breaking is induced radiatively.
- If discrete symmetry, $P = (-1)^{3B+L+2S}$ is imposed, lightest SUSY particle neutral and stable: Excellent candidate for cold Dark Matter.

Structure of Supersymmetric Gauge Theories

- The Standard Model is based on a Gauge Theory.
- A supersymmetric extension of the Standard Model has then to follow the rules of Supersymmetric Gauge Theories.
- These theories are based on two set of fields:
  - Chiral fields, that contain left handed components of the fermion fields and their superpartners.
  - Vector fields, containing the vector gauge bosons and their superpartners.
- Right-handed fermions are contained on chiral fields by means of their charge conjugate representation

$$(\psi_R)^C = \left(\psi^C\right)_L \tag{4}$$

- Higgs fields are described by chiral fields, with fermion superpartners

Field representations

- Field theories, describing the interactions of fundamental particles, are invariant under Lorentz transformations
- Lorentz Group is equivalent to $SU(2)_L \times SU(2)_R$, and therefore Lorentz group representations are labelled by two indeces (m,n)
- Spin 0-scalars have (0,0)
- Spin 1/2 fermions (1/2,0) (left-handed) and (0,1/2) (right-handed)
- Spin 1-vectors (1/2,1/2)

Graded Lie Algebra

- Coleman and Mandula demonstrated that the generators of symmetries of any fundamental field theory should reduce to the Poincare symmetry generators $P_\mu \;\; (1/2, 1/2)$ and $M_{\mu\nu} \;\; (0,1)+(0,1)$ plus generators that are scalars under Lorentz symmetries.
- Coleman and Mandula, however, considered only boson generators and never considered the possibility of generators that transform as fermions under the Lorentz group.
- Supersymmetry is the only extension of the possible set of fundamental symmetries of nature, including fermion generators.
- Since nature made use of all the other ones, shouldn't be natural to expect that it makes use also of Supersymmetry ?
- Let us then study what are the properties of a phenomenologically relevant supersymmetric theory. I will derive a set of rules to construct SUSY Lagrangians, through a fast tour in Superspace.

Generators of Supersymmetry (minimal set or N =1)

- Supersymmetry is a symmetry that relates boson to fermion degrees of freedom, $Q|F >= |B >$, $Q|B >= |F >$.
- The generators of supersymmetry are two component anticommuting spinors, Q_α, $\bar{Q}^{\dot{\alpha}}$, satisfying

$$\{Q_\alpha, Q_\beta\} = 0 = [P_\mu, Q_\alpha] \tag{5}$$

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu \tag{6}$$

where $\sigma^\mu = (I, \vec{\sigma})$, $\vec{\sigma}^\mu = (I, -\vec{\sigma})$, and σ^i are the Pauli matrices. As anticipated, space-time translations are part of the SUSY algebra.

- Two-spinors may be contracted to form Lorentz invariant quantities

$$\psi^\alpha \chi_\alpha = \psi^\alpha \epsilon_{\alpha\beta} \chi^\beta \tag{7}$$

Hamiltonian of Supersymmetric Theories

- Since there is a relation between the momentum operator and the SUSY genrators, one can compute the energy operator

$$H = \frac{1}{4}\left(Q_1 Q_1^\dagger + Q_1^\dagger Q_1 + Q_2 Q_2^\dagger + Q_2^\dagger Q_2\right)$$

- Two things may be concluded from here. First, the Hamiltonian operator is semidefinite positive.

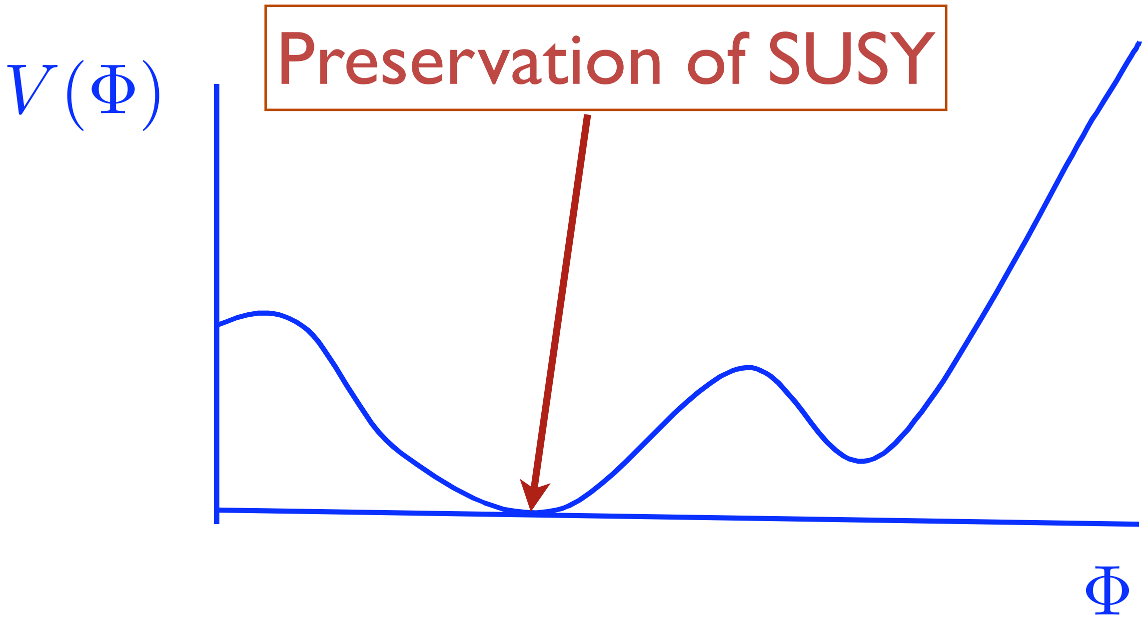
$$< \ H \ > = \ E \geq 0$$

- Second, if the theory is supersymmetric, then the vacuum state should be annihilated by supersymmetric charges

$$Q_\alpha |0 >= 0, \qquad Q^\dagger_{\dot\alpha} |0 >= 0 \qquad \Longrightarrow \qquad < 0|H|0 >= 0$$

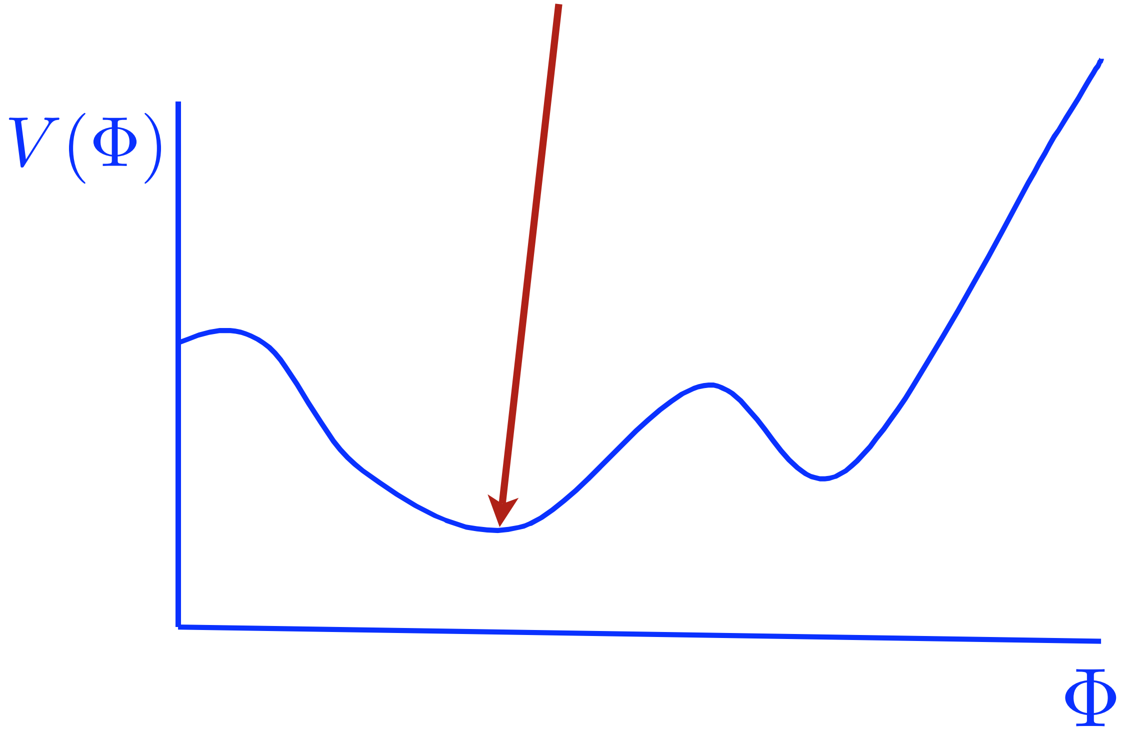
- So, the vacuum state energy is zero ! The vacuum energy is the order parameter for Supersymmetry breaking.

Effective Potential of a Supersymmetric Theory



Non-trivial Minimum could lead to the breakdown of gauge or global symmetries but SUSY is preserved, provided the value of the effective potential at the minimum is equal to zero, like in the Figure above.

Spontaneous breakdown of SUSY



If the Minimum of the Potential is at a value different from zero, then the vacuum state is not supersymmetric and SUSY has been broken spontaneously.
A massless fermion, the Goldstino, appears in the spectrum of the Theory.
In Supergravity (local supersymmetry) theories, this Goldstino appears as the longitudinal component of the Gravitino, of spin 3/2.

Four-component vs. Two-component fermions

- A Dirac Spinor is a four component object whose components are

$$\psi_D = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}; \qquad \psi_D^C = \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} \tag{8}$$

- A Majorana Spinor is a four component object whose components are

$$\psi_M = \begin{pmatrix} \chi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}; \qquad \psi_M^C = \psi_M \tag{9}$$

- Gamma Matrices

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}; \qquad \gamma^5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix} \tag{10}$$

- Observe that $\psi_{D,L} = \chi$; $\psi_{D,R} = \bar{\psi}$

- Usual Dirac contractions may be then expressed in terms of two component contractions.

$$\bar{\psi}_D = (\psi^\alpha \quad \bar{\chi}_{\dot{\alpha}}) \tag{11}$$

- For instance,

$$\bar{\psi}_D \, \psi_D = \psi\chi + h.c.; \tag{12}$$

$$\bar{\psi}_D \gamma^\mu \psi_D = \psi \bar{\sigma}^\mu \bar{\psi} + \bar{\chi}\sigma^\mu \chi \quad = \quad -\bar{\psi}\sigma^\mu \psi + \bar{\chi}\sigma^\mu \chi \tag{13}$$

Observe that Majorana particles lead to vanishing vector currents. Therefore, they must be neutral under electromagnetic interactions. Chiral currents don't vanish, $\bar{\psi}_D \gamma^\mu \gamma_5 \psi_D = -\bar{\psi}\sigma^\mu \psi - \bar{\chi}\sigma^\mu \chi$. They may couple to the Z-boson.

- Other relations may be found in the literature.

Superspace

- In order to describe supersymmetric theories, it proves convenient to introduce the concept of superspace.
- Apart from the ordinary coordinates x^μ, one introduces new anticommuting spinor coordinates θ^α and $\bar{\theta}_{\dot{\alpha}}$; $[\theta] = [\bar{\theta}] =$ -1/2.
- One can also define derivatives

$$\{\theta_\alpha, \theta_\beta\} = 0; \qquad \theta\theta\theta = 0; \qquad [\theta Q, \bar{\theta}\bar{Q}] = 2\theta\bar{\theta}\sigma^\mu P_\mu$$
$$\partial_\alpha = \frac{\partial}{\partial\theta^\alpha}; \qquad \partial_\alpha\theta^\beta = \delta_\alpha^\beta; \qquad \partial_\alpha(\theta^\beta\theta_\beta) = 2\theta_\alpha \tag{14}$$

Supersymmetry representation

- Supersymmetry is a particular translation in superspace, characterized by a Grassman parameter ξ.
- Supersymmetry generators may be given as derivative operators

$$Q_\alpha = i\left[-\partial_\theta - i\sigma^\mu\bar{\theta}\partial_\mu\right] \quad (15)$$

One can check that these diferential generators fulfill the SUSY algebra.

- Superspace allows to represent fermion and boson fields by the same superfield, by fields in superspace
- The operator

$$\bar{D} = -\partial_{\dot{\alpha}} + i\theta\sigma^\mu\partial_\mu$$

commutes with the supersymmetry transformations.

- So, if a field depends only on the variable $y^\mu = x^\mu - i\bar{\theta}\sigma^\mu\theta$, the supersymmetric transformation of it depends also on the y.

Chiral Fields $(\bar{D}\Phi = 0)$

- A generic scalar, chiral field is given by

$$\Phi(x, \theta, \bar{\theta} = 0) = A(x) + \sqrt{2}\,\theta\,\psi(x) + \theta^2 F(x)$$
$$\Phi(x, \theta, \bar{\theta}) = \exp(-i\partial_\mu \theta \sigma^\mu \bar{\theta})\;\Phi(x, \theta, \bar{\theta} = 0) \qquad (16)$$

The supersymmetric transformation of a chiral field is chiral.

- A, ψ and F are the scalar, fermion and auxiliary components.
- Under supersymmetric transformations, the components of chiral fields transform like

$$\delta A = \sqrt{2}\xi\psi, \qquad \delta F = -i\sqrt{2}\bar{\xi}\bar{\sigma}^\mu \partial_\mu \psi$$
$$\delta\psi = -i\sqrt{2}\sigma^\mu \bar{\xi}\partial_\mu A + \sqrt{2}\xi F \qquad (17)$$

- The F component transforms like a total derivative.

Properties of chiral superfields

- The product of two superfields is another superfield.
- For instance, the F-component of the product of two superfields Φ_1 and Φ_2 is obtained by collecting all the terms in θ^2, and is equal to

$$[\Phi_1 \Phi_2]_F = A_1 F_2 + A_2 F_1 - \psi_1 \psi_2 \tag{18}$$

- For a generic Polynomial function of several fields $P(\Phi_i)$, the result is

$$[P(\Phi)]_F = (\partial_{A_i} P(A)) F_i - \frac{1}{2} \left(\partial^2_{A_i A_j} P(A) \right) \psi_i \psi_j \tag{19}$$

- Finally, a single chiral field has dimensionality $[A] = [\Phi] = 1$, $[\psi]=$ 3/2 and $[F] = 2$. For $P(A)$, $[P(\Phi)]_F = [P(\Phi)] + 1$ ($[\theta] = [\bar{\theta}] =$ -1/2).

Expansion of Chiral Superfield

- In the above, we have only used the form of the chiral field at $\bar{\theta} = 0$.
- However, for many applications, the full expression of the chiral superfield is necessary. It is given by

$$\Phi(x, \theta, \bar{\theta}) = A(x) + i\partial^\mu A(x)\theta\sigma_\mu\bar{\theta} - \frac{1}{4}\partial^2 A(x)\theta^2\bar{\theta}^2 + \sqrt{2}\,\theta\psi(x) + i\frac{\theta^2}{2}\partial^\mu\psi(x)\sigma_\mu\bar{\theta} + F(x)\theta^2 \tag{20}$$

Vector Superfields

- Vector Superfields are generic hermitian fields. The minimal irreducible representations may be obtained by

$$V(x,\theta,\bar\theta) = -\left(\theta\sigma^\mu\bar\theta\right)V_\mu + i\theta^2\bar\theta\bar\lambda - i\bar\theta^2\theta\lambda + \frac{1}{2}\theta^2\bar\theta^2 D \qquad (21)$$

(Wess-Zumino gauge)

- Vector Superfields contain a regular vector field V_μ, its fermionic supersymmetric partner λ and an auxiliary scalar field D.
- Looking at the form of Q_α, it is easy to see that the D-component of a vector field transform like a total derivative.
- $D = [V] + 2$; $[V_\mu] = [V] + 1$; $[\lambda] = [V] + 3/2$. If V_μ describes a physical gauge field, then [V] = 0.

Superfield Strength and gauge transformations

- Similarly to $F_{\mu\nu}$ in the regular case, there is a field that contains the field strength. It is a chiral field, derived from V ($W = -\bar{D}\bar{D}DV/4$), and it is given by

$$W^{\alpha}(x,\theta,\bar{\theta}=0) = -i\lambda^{\alpha} + (\theta\sigma_{\mu\nu})^{\alpha} F^{\mu\nu} + \theta^{\alpha} D - \theta^2 \left(\bar{\sigma}^{\mu}\mathcal{D}_{\mu}\bar{\lambda}\right)^{\alpha} \quad (22)$$

- Under gauge transformations, superfields transform like

$$\begin{aligned} \Phi &\rightarrow \exp(-ig\Lambda)\Phi, \qquad W_{\alpha} \rightarrow \exp(-ig\Lambda)W_{\alpha}\exp(ig\Lambda) \\ \exp(gV) &\rightarrow \exp(-ig\bar{\Lambda})\exp(gV)\exp(ig\Lambda) \end{aligned} \quad (23)$$

where Λ is a chiral field of dimension 0.

Towards a Supersymmetric Lagrangian

- The aim is to construct a Lagrangian, invariant under supersymmetry and under gauge transformations.
- One should remember, for that purpose, that both the F-component of a chiral field, as well as the D-component of a vector field transform under SUSY as a total derivative.
- One should also remember that, if renormalizability is imposed, then the dimension of all interaction terms in the Lagrangian

$$[\mathcal{L}_{\rm int}] \leq 4 \tag{24}$$

- On the other hand,

$$[\Phi] = 1, \qquad [W_\alpha] = 3/2, \qquad [V] = 0. \tag{25}$$

and one should remember that $[V]_D = [V] + 2$; $[\Phi]_F = [\Phi] + 1$.

Supersymmetric Lagrangian

- Once the above machinery is introduced, the total Lagrangian takes a particular simple form. The total Lagrangian is given by

$$\begin{aligned} \mathcal{L}_{\rm SUSY} &= \frac{1}{4g^2}\left(Tr[W^\alpha W_\alpha]_F + h.c.\right) + \sum_i \left(\bar{\Phi}\exp(gV)\Phi\right)_D \\ &+ \left([P(\Phi)]_F + h.c.\right) \end{aligned} \qquad (26)$$

where $P(\Phi)$ is the most generic dimension-three, gauge invariant, polynomial function of the chiral fields Φ, and it is called Superpotential. It has the general expression

$$P(\Phi) = c_i\Phi_i + \frac{m_{ij}}{2}\Phi_i\Phi_j + \frac{\lambda_{ijk}}{3!}\Phi_i\Phi_k\Phi_k \qquad (27)$$

- The D-terms of V^a and the F term of Φ_i do not receive any derivative contribution: Auxiliary fields that can be integrated out by equation of motion. (Integration of auxiliary fields: scalar potential)

Lagrangian in terms of Component Fields

- The above Lagrangian has the usual kinetic terms for the boson and fermion fields. It also contain generalized Yukawa interactions and contain interactions between the gauginos, the scalar and the fermion components of the chiral superfields.

$$
\begin{aligned}
\mathcal{L}_{\mathrm{SUSY}} &= (\mathcal{D}_\mu A_i)^\dagger \mathcal{D} A_i + \left(\frac{i}{2} \bar{\psi}_i \bar{\sigma}^\mu \mathcal{D}_\mu \psi_i + \mathrm{h.c.} \right) \\
&- \frac{1}{4} \left(G^a_{\mu\nu} \right)^2 + \left(\frac{i}{2} \bar{\lambda}^a \bar{\sigma}^\mu \mathcal{D}_\mu \lambda^a + \mathrm{h.c.} \right) \\
&- \left(\frac{1}{2} \frac{\partial^2 P(A)}{\partial A_i \partial A_j} \psi_i \psi_j - i\sqrt{2} g A_i^* T_a \psi_i \lambda^a + h.c. \right) \\
&- V(F_i, F_i^*, D^a) \qquad (28)
\end{aligned}
$$

- The last term is a potential term that depend only on the auxiliary fields

Notation Refreshment

- All standard matter fermion fields are described by their left-handed components (using the charge conjugates for right-handed fields) ψ_i
- All standard matter fermion superpartners are described the scalar fields A_i. There is one for each chiral fermion.
- Gauge bosons are inside covariant derivatives and in the $G_{\mu\nu}$ terms.
- Gauginos, the superpartners of the gauge bosons are described by the fermion fields λ_a. There is one Weyl fermion for each massless gauge boson.
- Higgs bosons and their superpartners are described as regular chiral fields. Their only distinction is that their scalar components acquire a v.e.v. and, as we will see, they are the only scalars with positive R-Parity.

Scalar Potential

$$V(F_i, F_i^*, D^a) \;=\; \sum_i F_i^* F_i \;+\; \frac{1}{2}\sum_a (D^a)^2 \tag{29}$$

where the auxiliary fields may be obtained from their equation of motion, as a function of the scalar components of the chiral fields:

$$F_i^* = -\frac{\partial P(A)}{\partial A_i}, \qquad\qquad D^a = -g\sum_i (A_i^* T^a A_i) \tag{30}$$

Observe that the quartic couplings are governed by the gauge couplings and that scalar potential is positive definite ! The latter is not a surprise. From the supersymmetry algebra, one obtains,

$$H = \frac{1}{4}\sum_{\alpha=1}^{2}\left(Q_\alpha^\dagger Q_\alpha + Q_\alpha Q_\alpha^\dagger\right) \tag{31}$$

- If for a physical state the energy is zero, this is the ground state.
- Supersymmetry is broken if the vacuum energy is non-zero !

Couplings

- The Yukawa couplings between scalar and fermion fields,

$$\frac{1}{2}\frac{\partial^2 P(A)}{\partial A_i \partial A_j}\psi_i \psi_j + h.c. \tag{32}$$

are governed by the same couplings as the scalar interactions coming from

$$\left(\frac{\partial P(A)}{\partial A_i}\right)^2 \tag{33}$$

- Similarly, the gaugino-scalar-fermion interactions, coming from

$$-i\sqrt{2} g A_i^* T_a \psi_i \lambda^a + h.c. \tag{34}$$

are governed by the gauge couplings.

- No new couplings ! Same couplings are obtained by replacing particles by their superpartners and changing the spinorial structure.

Trilinear coupling

- A most useful example of the relation between couplings is provided by trilinear (Yukawa) couplings. To avoid complications, let's treat the abelian case:

$$P[\Phi] = h_t H U Q \tag{35}$$

where H is a Higgs superfield.

- Fermion Yukawa:

$$h_t H \psi_U \psi_Q + h.c. \qquad\qquad h_t \left(H \bar{\psi}_R \psi_L + h.c. \right) \tag{36}$$

- Scalar Yukawas

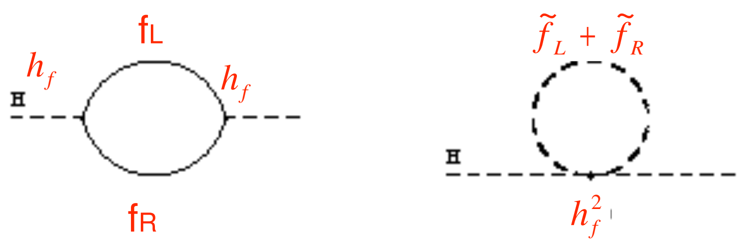
$$|h_t|^2 |H|^2 \left(|Q|^2 + |U|^2 \right) \tag{37}$$

- As anticipated, same couplings of the Higgs field to fermions and to scalar fields.

Higgs Mass Parameter Corrections in SUSY

One loop corrections to the Higgs mass parameter cancel if the couplings of scalars and fermions are equal to each other

$$\delta m_H^2 = \frac{N_C h_f^2}{16\pi^2}\left[-2\Lambda^2 + 3m_f^2 \log\left(\frac{\Lambda^2}{m_f^2}\right) + 2\Lambda^2 - 2m_{\tilde{f}}^2 \log\left(\frac{\Lambda^2}{m_{\tilde{f}}^2}\right)\right]$$



If supersymmetry is exact, there is always an additional, logarithmically divergent diagram, induced by the presence of Higgs-scalar trilinear couplings, that ensure the cancellation of the logarithmic term.

Properties of supersymmetric theories

- To each complex scalar A_i (two degrees of freedom) there is a Weyl fermion ψ_i (two degrees of freedom)
- To each gague boson V_μ^a, there is a gauge fermion (gaugino) λ^a.
- The mass eigenvalues of fermions and bosons are the same !
- Theory has only logarithmic divergences in the ultraviolet associated with wave-function and gauge-coupling constant renormalizations.
- Couplings in superpotential $P[\Phi]$ have no counterterms associated with them.
- The equality of fermion and boson couplings are essential for the cancellation of all quadratic divergences, at all oders in perturbation theory.

Supersymmetric Extension of the Standard Model

- Apart from the superpotential $P[\Phi]$, all other properties are directly determined by the gauge interactions of the theory.
- To construct the superpotential, one should remember that chiral fields contain only left-handed fields, and right-handed fields should be represented by their charge conjugates.
- SM right-handed fields are singlet under $SU(2)$. Their complex conjugates have opposite hypercharge to the standard one.
- There is one chiral superfield for each chiral fermion of the Standard Model.
- In total, there are 15 chiral fields per generation, including the six left-handed quarks, the six right-handed quarks, the two left-handed leptons and the right-handed charged leptons.

Minimal Supersymmetric Standard Model

SM particle	SUSY partner	G_{SM}
(S = 1/2)	(S = 0)	
$Q = (t, b)_L$	$(\tilde{t}, \tilde{b})_L$	(3,2,1/6)
$L = (\nu, l)_L$	$(\tilde{\nu}, \tilde{l})_L$	(1,2,-1/2)
$U = \left(t^C\right)_L$	$\tilde{t}^*_R$	($\bar{3}$,1,-2/3)
$D = \left(b^C\right)_L$	$\tilde{b}^*_R$	($\bar{3}$,1,1/3)
$E = \left(l^C\right)_L$	$\tilde{l}^*_R$	(1,1,1)
(S = 1)	(S = 1/2)	
B_μ	$\tilde{B}$	(1,1,0)
W_μ	$\tilde{W}$	(1,3,0)
g_μ	$\tilde{g}$	(8,1,0)

The Higgs problem

- Problem: What to do with the Higgs field ?
- In the Standard Model masses for the up and down (and lepton) fields are obtained with Yukawa couplings involving H and $H^\dagger$ respectively.
- Impossible to recover this from the Yukawas derived from $P[\Phi]$, since no dependence on $\bar{\Phi}$ is admitted.
- Another problem: In the SM all anomalies cancel,

$$\sum_{quarks} Y_i = 0; \qquad \sum_{left} Y_i = 0;$$
$$\sum_i Y_i^3 = 0; \qquad \sum_i Y_i = 0 \tag{38}$$

- In all these sums, whenever a right-handed field appear, its charge conjugate is considered.
- A Higgsino doublet spoils anomaly cancellation !

Solution to the problem

- Solution: Add a second doublet with opposite hypercharge.
- Anomalies cancel automatically, since the fermions of the second Higgs superfield act as the vector mirrors of the ones of the first one.
- Use the second Higgs doublet to construct masses for the down quarks and leptons.

$$P[\Phi] = h_u Q U H_2 + h_d Q D H_1 + h_l L E H_1 \tag{39}$$

- Once these two Higgs doublets are introduced, a mass term may be written

$$\delta P[\Phi] = \mu H_1 H_2 \tag{40}$$

- μ is only renormalized by wave functions of H_1 and H_2.

Higgs Fields

- Two Higgs fields with opposite hypercharge.

(S = 0)	(S = 1/2)	
H_1	$\tilde{H}_1$	(1,2,-1/2)
H_2	$\tilde{H}_2$	(1,2,1/2)

- Both Higgs fields acquire v.e.v. New parameter, $\tan\beta = v_2/v_1$.
- It is important to observe that the quantum numbers of H_1 are exactly the same as the ones of the lepton superfield L.
- This means that one can extend the superpotential $P[\Phi]$ to contain terms that replace H_1 by L.

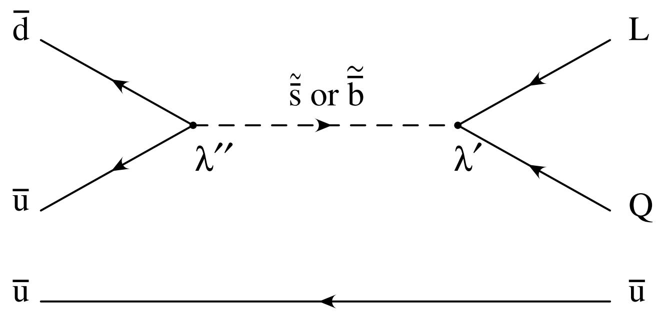
Baryon and Lepton Number Violation

- General superpotential contains, apart from the Yukawa couplings of the Higgs to lepton and quark fields, new couplings:

$$P[\Phi]_{\rm new} = \lambda' \, LQD + \lambda \, LLE + \lambda'' \, UDD \tag{41}$$

- Assigning every lepton chiral (antichiral) superfield lepton number 1 (-1) and every quark chiral (antichiral) superfield baryon number 1/3 (-1/3) one obtains :
  - Interactions in $P[\Phi]$ conserve baryon and lepton number.
  - Interactions in $P[\Phi]_{\rm new}$ violate either baryon or lepton number.
- One of the most dangerous consequences of these new interaction is to induce proton decay, unless couplings are very small and/or sfermions are very heavy.

Proton Decay



- Both lepton and baryon number violating couplings involved.
- Proton: Lightest baryon. Lighter fermions: Leptons

R-Parity

- A solution to the proton decay problem is to introduce a discrete symmetry, called R-Parity. In the language of component fields,

$$R_P = (-1)^{3B+2S+L} \tag{42}$$

- All Standard Model particles have $R_P = 1$.
- All supersymmetric partners have $R_P = -1$.
- All interactions with odd number of supersymmetric particles, like the Yukawa couplings induced by $P[\Phi]_{\rm new}$ are forbidden.
- Supersymmetric particles should be produced in pairs.
- The lightest supersymmetric particle is stable.
- Good dark matter candidate. Missing energy at colliders.

Lectures on Supersymmetry II
Phenomenology of low energy SUSY

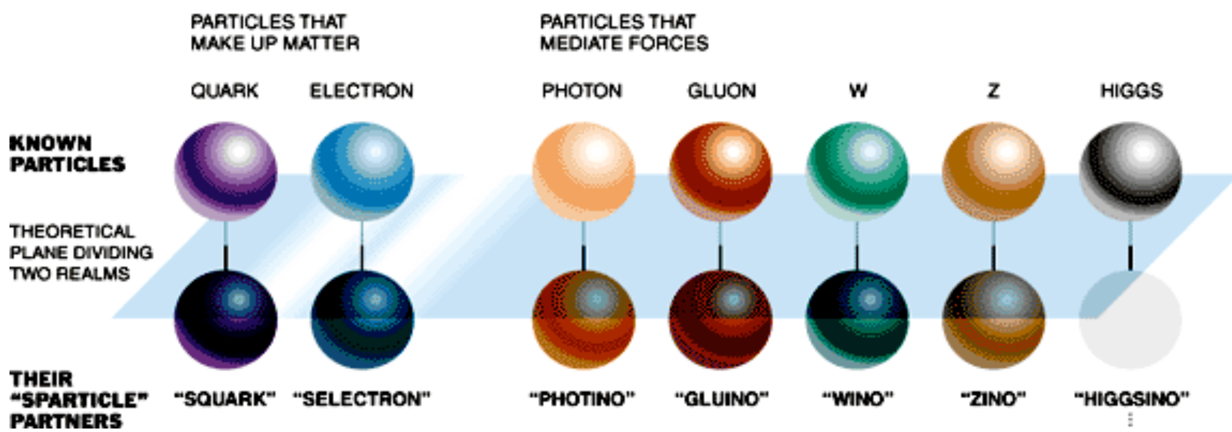
Carlos E.M. Wagner

Enrico Fermi Institute, University of Chicago
HEP Division, Argonne National Laboratory

Parma International School on Theoretical Physics, Univ. Parma, Sep. 2009

supersymmetry

fermions ⟷ **bosons**



Photino, Zino and Neutral Higgsino: Neutralinos

Charged Wino, charged Higgsino: Charginos

No new dimensionless couplings. Couplings of supersymmetric particles equal to couplings of Standard Model ones.
Two Higgs doublets necessary. Ratio of vacuum expectation values denoted by $\tan\beta$

Lagrangian in terms of Component Fields

- The supersymmetric Lagrangian has the usual kinetic terms for the boson and fermion fields. It also contain generalized Yukawa interactions and contain interactions between the gauginos, the scalar and the fermion components of the chiral superfields.

$$
\begin{aligned}
\mathcal{L}_{\rm SUSY} &= (\mathcal{D}_\mu A_i)^\dagger \mathcal{D} A_i + \left(\frac{i}{2}\bar{\psi}_i \bar{\sigma}^\mu \mathcal{D}_\mu \psi_i + \text{h.c.}\right) \\
&- \frac{1}{4}\left(G^a_{\mu\nu}\right)^2 + \left(\frac{i}{2}\bar{\lambda}^a \bar{\sigma}^\mu \mathcal{D}_\mu \lambda^a + \text{h.c.}\right) \\
&- \left(\frac{1}{2}\frac{\partial^2 P(A)}{\partial A_i \partial A_j}\psi_i\psi_j - i\sqrt{2} g A_i^* T_a \psi_i \lambda^a + h.c.\right) \\
&- V(F_i, F_i^*, D^a)
\end{aligned} \quad (1)
$$

- The last term is a potential term that depend only on the auxiliary fields

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H_2	$\tilde{H}_2$	(1,2,1/2)

- Both Higgs fields acquire v.e.v. New parameter, $\tan\beta = v_2/v_1$.
- One should use both Higgs doublets to give masses to quarks and leptons

$$P[\Phi] = h_u Q U H_2 + h_d Q D H_1 + h_l L E H_1 \tag{5}$$

- Once these two Higgs doublets are introduced, a mass term may be written

$$\delta P[\Phi] = \mu H_1 H_2 \tag{6}$$

Baryon and Lepton Number Violation

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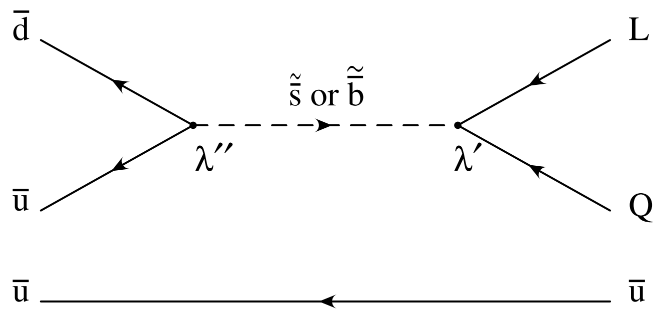

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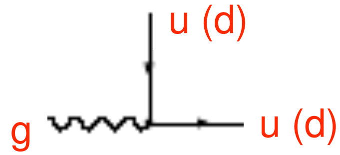
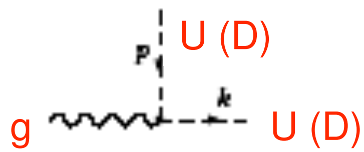
Feynman Rules for Supersymmetric Theories



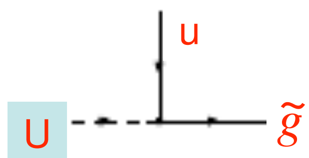
$$-ig_3 T^a \gamma^\mu$$

Start with SM couplings

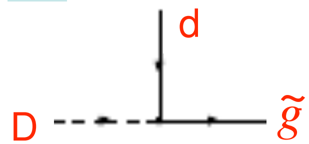


$$-ig_3 T^a (p+k)^\mu$$

Change fermion by scalars and gammas by momentum



$$-ig_3 T^a (c_U P_L + s_U P_R)$$

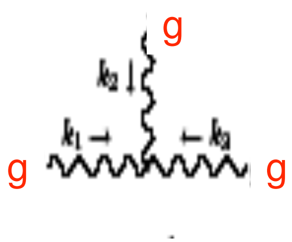


$$-ig_3 T^a (c_D P_L + s_D P_R)$$

Change one fermion by scalar and gluons by gluinos and gammas by constants (Yukawa Couplings). Extra factors are mixing angles that project mass eigenstates into gauge eigenstates.

Gluons and Gluinos

$$- g_3 f_{abc} \gamma^{\mu}$$



$$- g_3 f_{abc} \left[g^{\mu\nu} (k_1 - k_2)^{\lambda} + g^{\nu\lambda} (k_2 - k_3)^{\mu} + g^{\mu\lambda} (k_3 - k_1)^{\nu} \right]$$

Gluinos are strongly interacting particles and, unless very heavy, are one of the most copiously produced particles at hadron colliders

Scalar Interactions

- As we said before, the scalar potential may be obtained by adding the D-terms, that depend only on the gauge structure, with terms that depend on the square of the derivative of the superpotential.
- For the given superpotential, we get terms like

$$V_F = h_t \mu^* H_1^* Q U + h.c. \tag{8}$$

- Once the Higgs acquire a v.e.v., this induces a mixing between the right handed stop and left handed stop

$$- h_t \mu^* v_1 \tilde{t}_L \tilde{t}_R^* \tag{9}$$

- This will affect the masses, that, however, should be equal to the top-quark masses if supersymmetry is to be preserved ! What is going on ?

Preservation of Supersymmetry

- Let's look at the potential for the neutral Higgs bosons

$$V_{H^0} = |\mu|^2 \left(|H_1^0|^2 + |H_2^0|^2\right) + \frac{(g_1^2 + g_2^2)}{8} \left(|H_1^0|^2 - |H_2^0|^2\right)^2 \qquad (10)$$

- To preserve supersymmetry, we need the vacuum state to have zero energy.
- This may be only obtained, once the Higgs acquire v.e.v., if :

$$\mu = 0, \qquad\qquad \tan\beta = 1 \qquad (11)$$

- The potential presents a flat direction under these conditions.

Supersymmetry Breaking

- No supersymmetric particle have been seen: Supersymmetry is broken in nature
- Unless a specific mechanism of supersymmetry breaking is known, no information on the spectrum can be obtained.
- Cancellation of quadratic divergences:
  - Relies on equality of couplings and not on equality of the masses of particle and superpartners.
- Soft Supersymmetry Breaking: Give different masses to SM particles and their superpartners but preserves the structure of couplings of the theory.

Supersymmetry Breaking Parameters

Standard Model quark, lepton and gauge boson masses are protected by chiral and gauge symmetries.
Supersymmetric partners are not protected.
Explanation of absence of supersymmetric particles in ordinary experience/ high-energy physics colliders: Supersymmetric particles can acquire gauge invariant masses, as the one of the SM-Higgs.

Different kind of parameters:

Squark and slepton masses $m^2_{\tilde{q}}$, $m^2_{\tilde{l}}$
Gaugino (Majorana) masses M_i, $i = 1$-3
Trilinear scalar masses $(\tilde{f}^*_L \tilde{f}_R H_i)$ A_f, -μ^* (This last one comes from the scalar potential derived from the superpotential $|\partial P/\partial A_i|^2$.
They induce mixing between left and right sfermions.
Higgsino Mass μ and associated Higgs Mass Parameters
$|\mu|^2 + m^2_{H_i}$ (The first term may be derived from the superpotential).

Higgs Potential

- After supersymmetry breaking effects are considered, the Higgs potential reads

$$V(H_1, H_2) = m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 + m_3^2 (H_1^T i\tau_2 H_2 + h.c.) + \frac{\lambda_1}{2}\left(H_1^\dagger H_1\right)^2 + \frac{\lambda_2}{2}\left(H_2^\dagger H_2\right)^2 + \lambda_3 \left(H_1^\dagger H_1\right)\left(H_2^\dagger H_2\right) + \lambda_4 \left|(H_1^T i\tau_2 H_2)\right|^2$$

  where

$$\lambda_1 = \lambda_2 = \frac{g_1^2 + g_2^2}{4}, \qquad \lambda_3 = \frac{g_2^2 - g_1^2}{4}, \qquad \lambda_4 = -\frac{g_2^2}{2} \tag{12}$$

- This effective potential is valid at the scale of the SUSY particle masses.
- The value of the effective potential at low energies may be obtained by evolving the quartic couplings with their renormalization group equations.

Large $\tan\beta$ Limit

- This limit arises when, approximately, only one of the two v.e.v.'s is different from zero. To keep the top Yukawa coupling small, it should be v_2.

$$m_t = h_t v_2 \qquad\qquad m_b = h_b v_1 \tag{13}$$

- If one makes h_b large, of the order of h_t, $\tan\beta$ is about 50
- For this limit to happen $m_3^2 \simeq 0$.
- Then, the doublet H_2 contains the Goldstone modes and the "physical" SM-like Higgs boson, while H_1 contains a scalar, a pseudoscalar and a charged Higgs boson.
- Physical Higgs mass ($m_2^2 = -M_Z^2/2$)

$$m_h^2 = 2\lambda_2 v^2 = M_Z^2 \tag{14}$$

Evolution of Paramters

- In general, the parameters that one measures at low energies (large distances) are not the fundamental ones, but they are modified by quantum corrections.
- For instance, if you put a charge into the vacuum state, it will polarize the vacuum by inducing the production of virtual particles and antiparticles, which "screen" the original charge.
- This also happens with other couplings and also with mass parameters. There are equations, called renormalization group equations that allow to relate the fundamental parameters to the ones at low energies.

Higgs Boson Mass at large values of $\tan\beta$

- The RG evolution of λ_2 is given by

$$\frac{d\lambda_2}{dt} \simeq -\frac{3}{8\pi^2}\left[\lambda_2^2 + \lambda_2 h_t^2 - h_t^4\right] \tag{15}$$

  with $t = log(M_{SUSY}^2/Q^2)$.

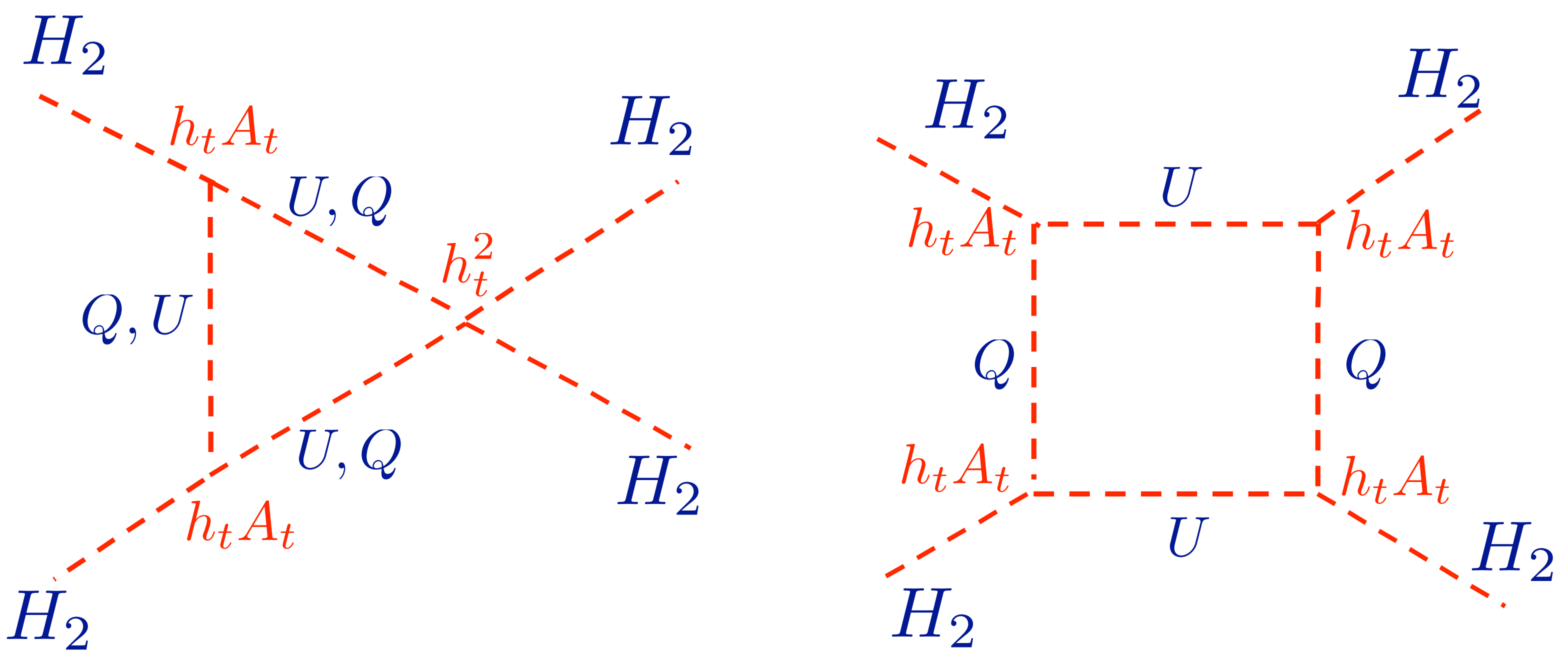
- For large values of $\tan\beta = v_2/v_1$, the Higgs H_2 is the only one associated with electroweak symmetry breaking.

- The Higgs boson mass is approximately given by $m_h^2 = 2\lambda_2 v^2$

$$m_h^2 \simeq M_Z^2 + \frac{3m_t^4}{4\pi^2 v^2}\left[\log\left(\frac{M_{SUSY}^2}{m_t^2}\right) + \frac{A_t^2}{M_{SUSY}^2}\left(1 - \frac{A_t^2}{12M_{SUSY}^2}\right)\right] \tag{16}$$

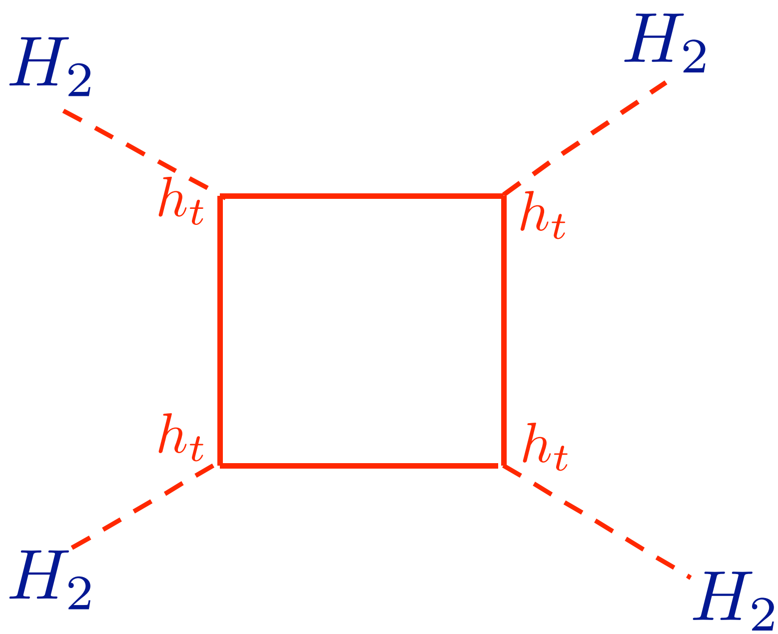
- The first term comes from the SUSY contribution. The logarithmic term comes from the RG evolution, while the A_t dependence comes from threshold effects at M_{SUSY}.

Diagrams Contributing to the Quartic Coupling



This diagrams provide the finite threshold corrections after decoupling of the top quark superpartners.

Top Quark Contribution to the Higgs Quartic Coupling



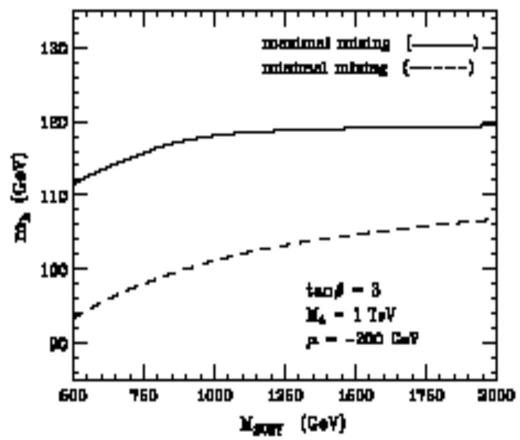
This diagram provides the dominant logarithmic contribution below the stop quark mass scale.

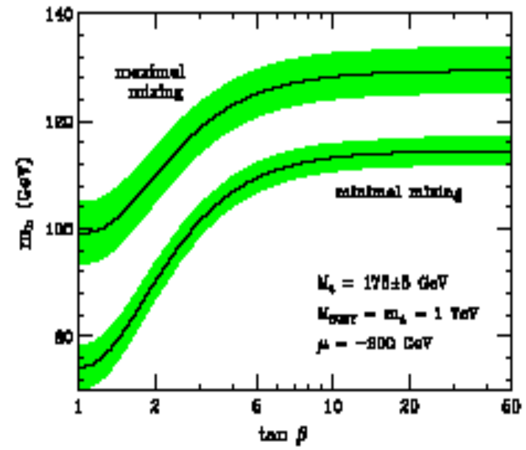
Stop Mass Matrix

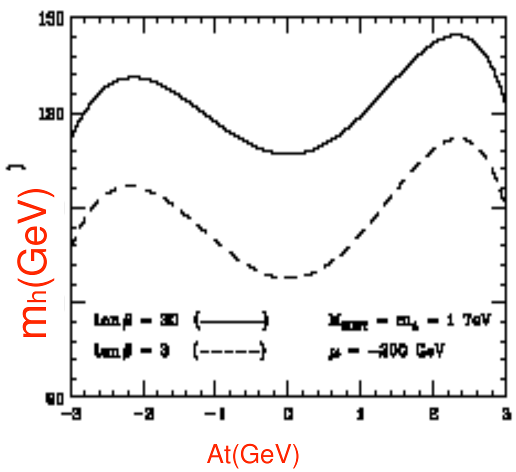
- The stop, and other squarks, acquire masses that are controlled by the supersymmetry breaking parameters.
- Once the Higgs acquires a v.e.v., the mass matrix is

$$M_{\tilde{t}}^2 = \begin{bmatrix} m_Q^2 + m_t^2 & m_t(A_t - \mu^*/\tan\beta) \\ m_t(A_t^* - \mu/\tan\beta) & m_U^2 + m_t^2 \end{bmatrix} \qquad (17)$$

- In general, the existence of A_t and μ denote couplings of the stops to the Higgs bosons, that induce finite corrections to the quartic couplings.







- m_t^4 enhancement
- logarithmic sensitivity to $m_{\tilde{t}_i}$
- depend. on $\tilde{t}$-mixing X_t

$\Longrightarrow$ max. value $X_t \sim \sqrt{6} M_S$

Carena, Haber, Hollik, Heinemeyer, Weiglein, C.W. '00
Heinemeyer, Hollik, Weiglein'02
Degrassi, Slavich, Zwirner '02

$M_{SUSY} \equiv M_Q = M_U = M_D$ if $M_{SUSY} \gg m_t \rightarrow M_S^2 \simeq M_{SUSY}^2$

- at 2 loops $\rightarrow M_{\tilde{g}}$ dependence

Higgs Spectrum

- The two Higgs doublets carry eight real scalar degrees of freedom.
- Three of them are the charged and CP-odd Goldstone bosons that are absorbed in the longitudinal components of the W and the Z.
- Five Higgs bosons remain: Two CP-even, one CP-odd, neutral bosons, and a charged Higgs boson (two degrees of freedom).
- Generically, the electroweak breaking sector (Goldstones and real Higgs) is contained in the combination of doublets

$$\Phi = \cos\beta H_1 + \sin\beta i\tau_2 H_2^*, \tag{18}$$

while the orthogonal combination contains the other Higgs bosons. Their masses are:

$$m_H^2 \simeq m_A^2, \qquad m_{H^\pm}^2 \simeq m_A^2 + M_W^2 \tag{19}$$

with $m_A^2 = m_1^2 + m_2^2$. These relations are preserved, in a good approximation, after loop-effects.

Masses of the Higgs boson particles

- There is a light Higgs boson, which, for supersymmetry breaking masses of the order of 1 TeV, has a mass

$$m_h \leq 130 \text{ GeV}$$

- This Higgs boson is the "real Higgs", in the sense that it is the one connected to the mechanism of electroweak symmetry breaking. It has SM -like properties.

- Then, there are two other neutral Higgs bosons, one being CP-odd and the other CP-even. Their mass is controled by the parameter m_A

- This parameter, which controls also the charged Higgs mass is governed by supersymmetry breaking masses $(m_A^2 = m_1^2 + m_2^2)$ and therefore can be very large (but it could be around the corner, too)

Gaugino/Higgsino Mixing

- Just like the gauge boson mixes with the Goldstone modes of the theory after spontaneous breakdown of the gauge symmetry, gauginos mix with the Higgsinos.
- Mixing comes from the interaction $\sqrt{2} g A_i^* T_a \psi_i \lambda^a$, when one takes $A_i \equiv H_i$, and $\lambda^a \equiv \tilde{W}^a$, $\tilde{B}$, and $\psi_i = \tilde{H}_i$.
- Charged Winos, $\tilde{W}_1 \pm i \tilde{W}_2$, mix with the charged components of the Higgsinos $\tilde{H}_{1,2}$. The mass eigenstates are called charginos $\tilde{\chi}^{\pm}$.
- Neutral Winos and Binos, $\tilde{B}$, $\tilde{W}_3$ mix with the neutral components of the Higgsinos. The mass eigenstates are called neutralinos, $\tilde{\chi}^0$.
- Charginos form two Dirac massive fields. Neutralinos give four massive Majorana states.

Chargino Mass matrix

Lets take, for instance, the chargino mass matrix in the basis of Winos and Higgsinos, $(\tilde{W}^+, \tilde{H}_2^+)$ and $(\tilde{W}^-, \tilde{H}_1^-)$, with $\tilde{W}^\pm = \tilde{W}^1 \pm i\tilde{W}^2$. The mixing term is proportional to the weak coupling and the Higgs v.e.v.'s

$$M_{\tilde{\chi}^\pm} = \begin{bmatrix} M_2 & g_2 v_2 \\ g_2 v_1 & \mu \end{bmatrix} \tag{20}$$

Here, M_2 is the soft breaking mass term of the Winos and μ is the Higgsino mass parameter.

- The eigenstates are two Dirac, charged fermions (charginos).
- If μ is large, the lightest chargino is a Wino, with mass M_2, and its interactions to fermion and sfermions are governed by gauge couplings.
- If M_2 is large, the lightest chargino is a Higgsino, with mass μ, and the interactions are governed by Yukawa couplings.

Neutralino Mass Matrix

Similarly, for neutralinos in the basis of Binos, Winos and Higgsinos

$$M_{\tilde{\chi}^0} = \begin{bmatrix} M_1 & 0 & -g_1 v_1/\sqrt{2} & g_1 v_2/\sqrt{2} \\ 0 & M_2 & g_2 v_1/\sqrt{2} & -g_2 v_2/\sqrt{2} \\ -g_1 v_1/\sqrt{2} & g_2 v_1/\sqrt{2} & 0 & -\mu \\ g_1 v_2/\sqrt{2} & -g_2 v_2/\sqrt{2} & -\mu & 0 \end{bmatrix} \qquad (21)$$

- The eigenstates are four Majorana particles.
- If the theory proceeds from a GUT, there is a relation between M_2 and M_1, $M_2 \simeq \alpha_2(M_Z)/\alpha_1(M_Z) M_1 \simeq 2M_1$.
- So, if μ is large, the lightest neutralino is a Bino (superpartner of the hypercharge gauge boson) and its interactions are governed by g_1.
- This tends to be a good dark matter candidate.

Counting degrees of Freedom

- The charginos are two Dirac particles, with eight degrees of freedom, and are an admixture of the superpartners of the charged gauge bosons and Higgs bosons.
- The boson sector has the $W^{\pm}$, that has six degrees of freedom, plus the charged Higgs, with two degrees of freedom. Observe that before electroweak symmetry breaking there is no mixing and the numbers are four and four, respectively.
- Neutralinos are four Majorana particles, and have eight degrees of freedom.
- There are five in the neutral gauge bosons (photon and Z) plus three in the neutral Higgs bosons. Again, before electroweak symmetry breaking, the numbers were four and four.

Structure of Supersymmetry Breaking Parameters

- Although there are few supersymmetry breaking parameters in the gaugino and Higgsino sector, there are many in the scalar sector.
- For instance, there are 45 scalar states and all these scalar masses might be different. In addition, one can add complex scalar mass parameters that mix squark and sleptons of different generations: $\mathcal{L}_{\rm mix} = m^2_{ij}\tilde{f}^*_i\tilde{f}_j + h.c.$
- In addition, one can add A-terms that also mix squarks and sleptons of different generations.
- In general, in the presence of such terms, if the scalar masses are of the order of the weak scale, one can induce contributions to flavor changing neutral currents by interchanging gauginos and scalars.
- This leads to problematic phenomenological consequences.

Flavor Changing Neutral Currents

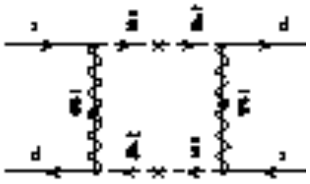
- Two particularly constraining examples of flavor changing neutral currents induced by off-diagonal soft supersymmetry breaking parameters

- Contribution to the mixing in the Kaon sector, as well as to the rate of decay of a muon into an electron and a photon.

- While the second is in good agreement with the SM predictions, the first one has never been observed.

- Rate of these processes suppressed as a power of supersymmetric particle masses and they become negligible *only if relevant masses are larger than 10 TeV*

(a)



(b)

Solution to the Flavor Problem

- There are two possible solutions to the flavor problem
- The first one is to push the masses of the scalars, in particular to the first and second generation scalars, to very large values, larger than *or of the order of 10 TeV.*
- Some people have taken the extreme attitude of pushing them to values of order of the GUT scale. This is fine, but supersymmetry is then broken in a hard way and the solution to the hierarchy problem is lost.
- A second possibility is to demand that the scalar mass parameters are approximately flavor diagonal in the basis in which the fermions mass matrices are diagonal. All flavor violation is induced by either CKM mixing angles, or by very small off-diagonal mass terms.
- This latter possibility is a most attractive one because it allows to keep SUSY particles with masses of the order of the weak scale.

Minimal Supergravity Model

- The simplest possibility is the case in which all scalar masses are universal and flavor independent at a certain scale.
- In the minimal supergravity model, for example, one assumes that all scalars acquire a common mass m_0^2 at the Grand Unification scale
- In addition, since gauginos belong to the same adjoint representation, one assumes that all gauginos acquire a common mass $M_{1/2}$ at the GUT scale
- These two parameters must be complemented with a value of the parameter μ at M_{GUT}.
- For the Higgs sector it is assumed that $m_i^2 = |\mu|^2 + m_{H_i}^2$, with $m_{H_i} = m_0$.
- Finally, all the trilinear parameters A_{ijk} are assume to take a common value A_0.

Renormalization Group Evolution

- One interesting thing is that the gaugino masses evolve in the same way as the gauge couplings:
  $d(M_i/\alpha_i)/dt = 0, \quad dM_i = -b_i \alpha_i M_i/4\pi, \qquad d\alpha_i/dt = -b_i \alpha_i^2/4\pi$

- The scalar fields masses evolve in a more complicated way.
  $4\pi dm_i^2/dt = -C_a^i 4M_a^2 \alpha_a + |Y_{ijk}|^2[(m_i^2 + m_j^2 + m_k^2 + A_{ijk}^2)]/4\pi$
- There is a positive contribution coming from the gaugino masses and a negative contribution proportional to the Yukawa couplings.
- Colored particles are affected by positive, strongly coupled corrections and tend to be the heaviest ones.
- Weakly interacting particles tend to be lighter, particular those affected by large Yukawas.
- There scalar field H_2 is both weakly interacting and couples with the top quark Yukawa. Its mass naturally becomes negative.

Low Energy Masses in Minimal Supergravity Model

All scalars acquire a common mass m_0^2 at the Grand Unification scale
All gauginos acquire a common mass $M_{1/2}$ at the GUT scale
Masses evolve differently under R.G.E. At low energies,

Squark Masses: $m_{\tilde{Q}}^2 \simeq m_0^2 + 6\ M_{1/2}^2$
Left-Slepton Masses $m_{\tilde{L}}^2 \simeq m_0^2 + 0.5\ M_{1/2}^2$
Right-Slepton Masses $m_{\tilde{E}}^2 \simeq m_0^2 + 0.15\ M_{1/2}^2$

Wino Mass $M_2 = 0.8\ M_{1/2}$.
Gluino Mass $M_3 = \frac{\alpha_3}{\alpha_2} M_2$
Bino Mass $M_1 = \frac{\alpha_1}{\alpha_2} M_2$

Lightest SUSY particle tends to be a Bino.

Electroweak Symmetry Breaking

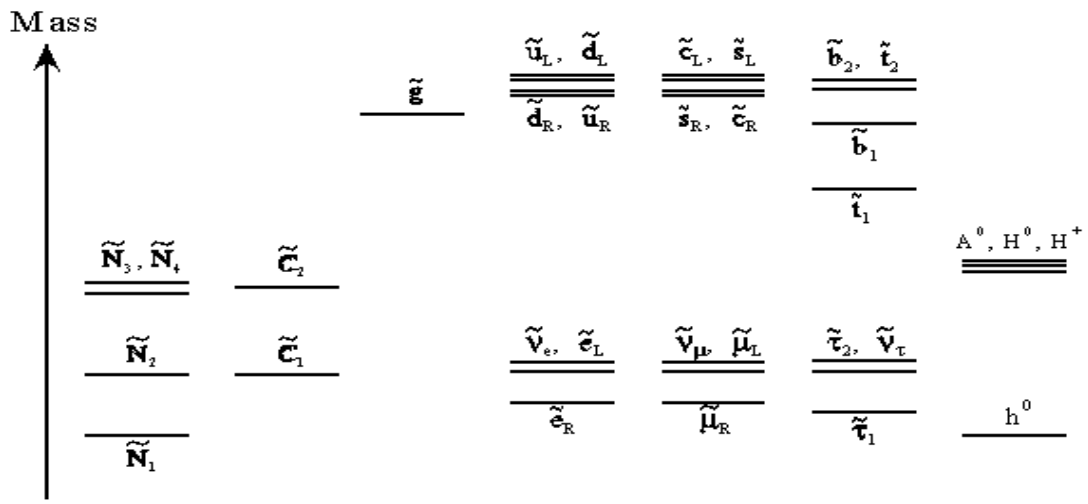
- The above relations apply to most squarks and leptons, but not to the Higgs particles and the third generation squarks.
- The renormalization group equations of these mass parameters include negative corrections proportional to the square of the large top Yukawa coupling.
- In particlular, the H_2 Higgs mass parameter m_2^2, is driven to negative values due to the influence of the top quark Yukawa coupling.
- Electroweak symmetry breaking is induced by the large top mass !
- Also the superpartners of the top quark tend to be lighter than the other squarks. This effect is more pronounced if $M_{1/2}$ is small.

SUSY Spectrum from Universal Boundary Conditions at M_{GUT}

Coloured, strongly interacting particles tend to be heavier than weakly interacting particles.

Lightest Supersymmetric Particle is lightest Neutralino.

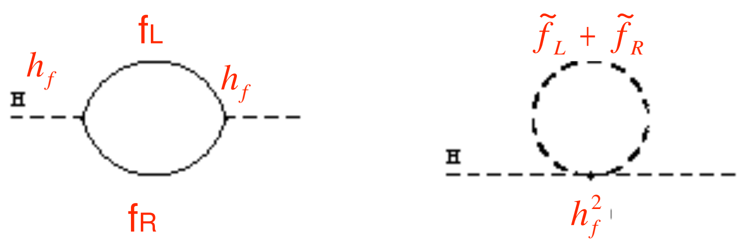
Squarks and sleptons of first two generations are degenerate. Small splitting in the slepton sector. Third generation squarks are Typically lighter.



Higgs Mass Parameter Corrections in SUSY

One loop corrections to the Higgs mass parameter cancel if the couplings of scalars and fermions are equal to each other

$$\delta m_H^2 = \frac{N_C h_f^2}{16\pi^2}\left[-2\Lambda^2 + 3m_f^2 \log\left(\frac{\Lambda^2}{m_f^2}\right) + 2\Lambda^2 - 2m_{\tilde{f}}^2 \log\left(\frac{\Lambda^2}{m_{\tilde{f}}^2}\right)\right]$$



If supersymmetry is exact, there is always an additional, logarithmically divergent diagram, induced by the presence of Higgs-scalar trilinear couplings, that ensure the cancellation of the logarithmic term.

If SUSY exists, many of its most important motivations demand some SUSY particles at the TeV range or below

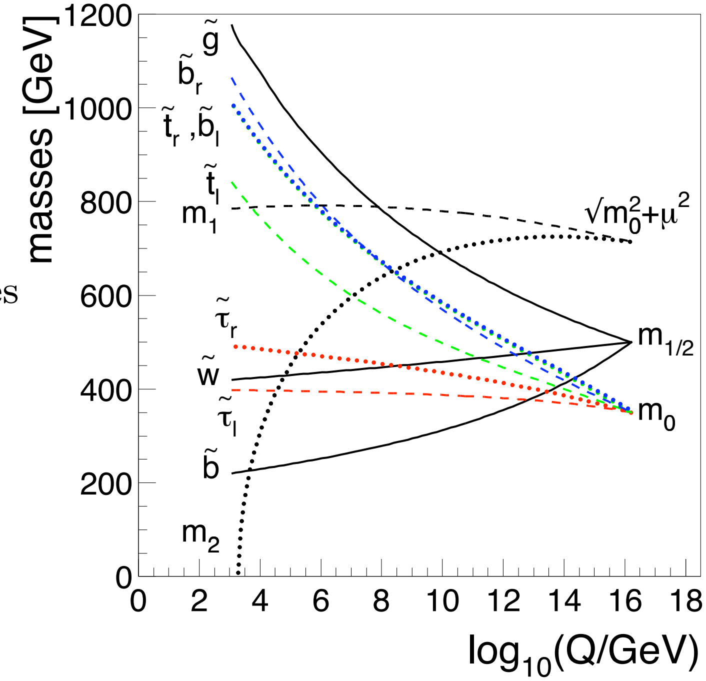
⋆ Solve hierarchy/naturalness problem by having $\Delta m^2 \simeq \mathcal{O}(v^2)$

SUSY breaking scale must be at or below 1 TeV if SUSY is associated with EWSB scale !

⋆ EWSB is radiatively generated

In the evolution of masses from high energy scales
⟶ a negative Higgs mass parameter is induced via radiative corrections

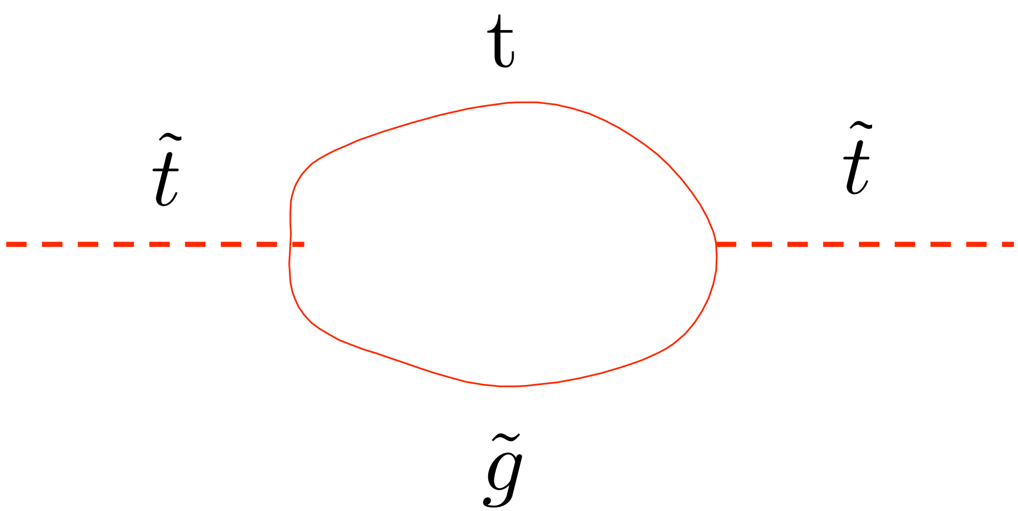
⟹ *important top quark effects!*



Reasons for the negative mass of the Higgs

- Higgs is affected by the top quark Yukawa coupling, but so are the stops. So, why does the Higgs mass becomes negative and not the stop ones ?
- There are three main reasons. First, the stops receive a positive contributions coming from their quark-gluino interactions, which is proportional to the gluino mass. This is model dependent, since the gluino mass might be very small.
- The second, most important reason, is that the Higgs couples to three Dirac fermions (three colors) and their superpartners, while the right-handed stop of a given color couples to only two (due to the doublet structure). Finally the left-handed stop of a given color couples to only one Dirac field. This 3:2:1 relation makes the negative contribution to the Higgs more important than the one of the stops.
- Finally, there is the m_{12}^2 factor that acts like an external magnetic field in a spin system and induces the presence of non-vanishing Higgs v.e.v.'s even if the masses would be positive.

Gluino Contributions to the Stop Masses



$$\delta m_{\tilde{t}}^2 \propto \alpha_3 \, M_{\tilde{g}}^2$$

Comments on Minimal Supergravity Spectrum

- The previously presented spectrum depends strongly on the condition of equality of sfermion and gaugino masses at the GUT scale.

- Setting, for instance, different masses for particles of different quantum numbers at the GUT scale could lead to a very different spectrum.

- In general, very little is known about the supersymmetry breaking parameters and one should NOT make conclusions about the Tevatron and/or LHC reach for SUSY based on strong assumptions about them.

- In particular, although it is clear that the LHC has a larger reach, the Tevatron one is not at all negligible, and one should be open to the possibility of a SUSY discovery before the start of the LHC !

Unification of Couplings

- The value of gauge couplings evolve with scale according to the corresponding RG equations:

$$\frac{1}{\alpha_i(Q)} = \frac{b_i}{2\pi} \ln\left(\frac{Q}{M_Z}\right) + \frac{1}{\alpha_i(M_Z)} \tag{8}$$

- Unification of gauge couplings would occur if there is a given scale at which couplings converge.

$$\frac{1}{\alpha_3(M_Z)} = \frac{b_3 - b_2}{b_1 - b_2}\frac{1}{\alpha_1(M_Z)} + \frac{b_3 - b_1}{b_2 - b_1}\frac{1}{\alpha_2(M_Z)} \tag{9}$$

- This leads to a relation between $\alpha_3(M_Z)$ and $\sin^2\theta_W(M_Z) = \alpha_1^{SM}/\left(\alpha_1^{SM} + \alpha_2^{SM}\right)$.

Rules to compute the beta-functions

- The one-loop beta-functions for the $U(1)$ and $SU(N)$ gauge couplings are given by ($Q = T_3 + Y$),

$$\begin{aligned} \frac{5}{3} b_1 &= -\frac{2}{3} \sum_f y_f^2 - \frac{1}{3} \sum_s y_s^2 \\ b_N &= \frac{11\,N}{3} - \frac{n_f}{3} - \frac{n_S}{6} - \frac{2N}{3} n_A \end{aligned} \tag{10}$$

- In the above, $y_{f,s}$ are the hypercharges of the charged chiral fermion and scalar fields, $n_{f,s}$ are the number of fermions and scalars in the fundamental representation of $SU(N)$, n_A are fermions in the adjoint representation and the factor $5/3$ is just a normalization factor, so that over one generation

$$Tr\left[T^3 T^3\right] = \frac{3}{5} Tr\left[y_f^2\right] \tag{11}$$

- The coupling g_1 is not asymptotically free, but becomes strong at scales far above M_{Pl}, where the effective theory description breaks down anyway.
- A full generation contributes to the same amount to all β-functions. It does not affect the unification conditions. ($\beta_{\rm gen}^{SM} = -4/3$; $\beta_{\rm gen}^{\rm SUSY} = -2$.)
- Only incomplete $SU(5)$ representations affect one-loop unification.

$$
\begin{aligned}
b_1^{SM} &= -41/10, \quad b_2^{SM} = 19/6, \quad b_3^{SM} = 7. \\
b_1^{\rm SUSY} &= -33/5, \quad b_2^{\rm SUSY} = -1, \quad b_3^{\rm SUSY} = 3.
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
M_G = M_Z \exp\left[\left(\frac{1}{\alpha_1(M_Z)} - \frac{1}{\alpha_2(M_Z)}\right)\frac{2\pi}{b_2 - b_1}\right] \\
\frac{1}{\alpha_3(M_Z)} = \frac{b_3 - b_1}{b_2 - b_1}\frac{1}{\alpha_2(M_Z)} + \frac{b_3 - b_2}{b_1 - b_2}\frac{1}{\alpha_1(M_Z)}
\end{aligned} \tag{13}
$$

Results

$b_1 = -6 - 3N_H/5, \qquad b_2 = -N_H, \qquad b_3 = 3$

$1/\alpha_1(M_Z) \simeq 60 \qquad 1/\alpha_2(M_Z) \simeq 30$

In the above, N_H is the number of pair of Higgs doublets. With these results we can compute the strong gauge coupling at M_Z

$$\frac{1}{\alpha_3(M_Z)} = \frac{15 - 7N_H}{25 + 3N_H} 30 \tag{1}$$

The result depends strongly on the number of Higgs doublets. For N_H equal to zero, we get $\alpha_3(M_Z) = 1/18 \simeq 0.05$. For $N_H = 1$, instead, we get $\alpha_3(M_Z) \simeq 0.120$!! Excellent prediction !

For $N_H = 2$ we get, again, a very bad result $\alpha_3(M_Z) \simeq 1$.

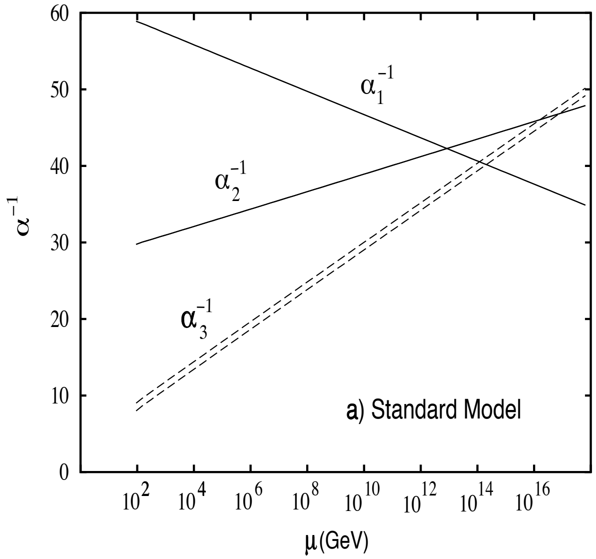
Finally, for $N_H = 1$,

$M_G = M_Z \exp(30 \times 2\pi \times (5/28)) \simeq 10^{16}\text{GeV}$

Grand Unification scale is very close to the Planck scale, suggesting a Unified description of particle interactions.

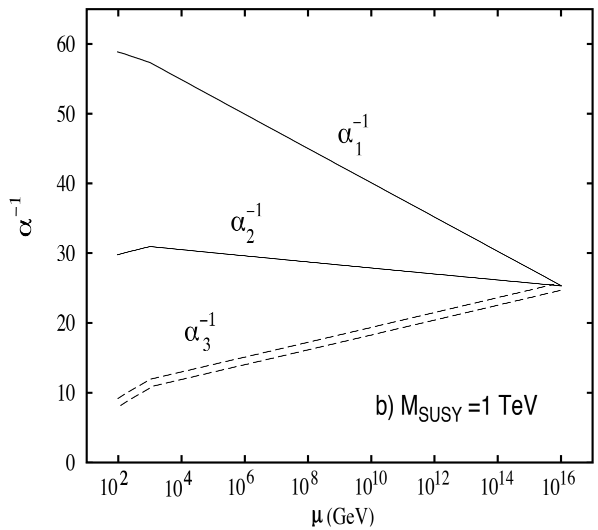
SM:

Couplings tend to converge at high energies, but unification is quantitatively ruled out.



MSSM:

Unification at $\alpha_{GUT} \simeq 0.04$ and $M_{GUT} \simeq 10^{16}$ GeV.



Experimentally, $\alpha_3(M_Z) \simeq 0.118 \pm 0.004$

Bardeen, Carena, Pokorski & C.W.

in the MSSM: $\alpha_3(M_Z) = 0.127 - 4(\sin^2\theta_W - 0.2315) \pm 0.008$

Remarkable agreement between Theory and Experiment!!

Lectures on Supersymmetry III
More on Phenomenology of SUSY

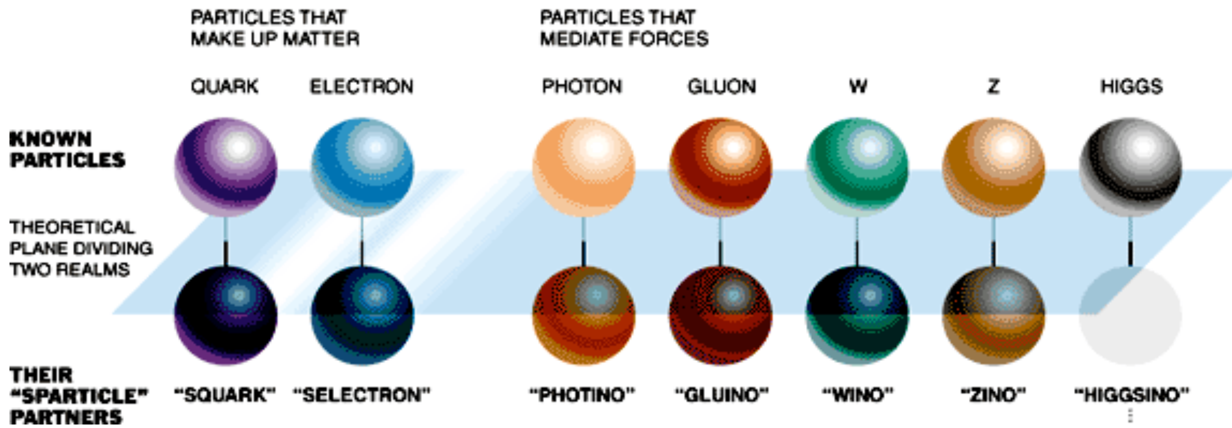
Carlos E.M. Wagner

Enrico Fermi Institute, University of Chicago
HEP Division, Argonne National Laboratory

Parma International School on Theoretical Physics, Univ. Parma, Sep. 2009

supersymmetry

fermions ⟷ **bosons**



Photino, Zino and Neutral Higgsino: Neutralinos

Charged Wino, charged Higgsino: Charginos

No new dimensionless couplings. Couplings of supersymmetric particles equal to couplings of Standard Model ones.
Two Higgs doublets necessary. Ratio of vacuum expectation values denoted by $\tan\beta$

Lagrangian in terms of Component Fields

- The supersymmetric Lagrangian has the usual kinetic terms for the boson and fermion fields. It also contain generalized Yukawa interactions and contain interactions between the gauginos, the scalar and the fermion components of the chiral superfields.

$$\begin{aligned}
\mathcal{L}_{\rm SUSY} &= (\mathcal{D}_\mu A_i)^\dagger \mathcal{D} A_i + \left(\frac{i}{2}\bar{\psi}_i \bar{\sigma}^\mu \mathcal{D}_\mu \psi_i + \text{h.c.}\right) \\
&- \frac{1}{4}\left(G^a_{\mu\nu}\right)^2 + \left(\frac{i}{2}\bar{\lambda}^a \bar{\sigma}^\mu \mathcal{D}_\mu \lambda^a + \text{h.c.}\right) \\
&- \left(\frac{1}{2}\frac{\partial^2 P(A)}{\partial A_i \partial A_j}\psi_i\psi_j - i\sqrt{2} g A_i^* T_a \psi_i \lambda^a + h.c.\right) \\
&- V(F_i, F_i^*, D^a) \qquad (1)
\end{aligned}$$

- The last term is a potential term that depend only on the auxiliary fields

Minimal Supersymmetric Standard Model

SM particle	SUSY partner	G_{SM}
(S = 1/2)	(S = 0)	
$Q = (t, b)_L$	$(\tilde{t}, \tilde{b})_L$	(3,2,1/6)
$L = (\nu, l)_L$	$(\tilde{\nu}, \tilde{l})_L$	(1,2,-1/2)
$U = \left(t^C\right)_L$	$\tilde{t}^*_R$	($\bar{3}$,1,-2/3)
$D = \left(b^C\right)_L$	$\tilde{b}^*_R$	($\bar{3}$,1,1/3)
$E = \left(l^C\right)_L$	$\tilde{l}^*_R$	(1,1,1)
(S = 1)	(S = 1/2)	
B_μ	$\tilde{B}$	(1,1,0)
W_μ	$\tilde{W}$	(1,3,0)
g_μ	$\tilde{g}$	(8,1,0)

Supersymmetry Breaking Parameters

Standard Model quark, lepton and gauge boson masses are protected by chiral and gauge symmetries.
Supersymmetric partners are not protected.
Explanation of absence of supersymmetric particles in ordinary experience/ high-energy physics colliders: Supersymmetric particles can acquire gauge invariant masses, as the one of the SM-Higgs.

Different kind of parameters:

Squark and slepton masses $m_{\tilde{q}}^2$, $m_{\tilde{l}}^2$
Gaugino (Majorana) masses $M_i, \quad i = 1\text{-}3$
Trilinear scalar masses $(\tilde{f}_L^* \tilde{f}_R H_i)$ A_f, -μ^* (This last one comes from the scalar potential derived from the superpotential $|\partial P/\partial A_i|^2$.
They induce mixing between left and right sfermions.
Higgsino Mass μ and associated Higgs Mass Parameters
$|\mu|^2 + m_{H_i}^2$ (The first term may be derived from the superpotential).

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- The simplest possibility is the case in which all scalar masses are universal and flavor independent at a certain scale.
- In the minimal supergravity model, for example, one assumes that all scalars acquire a common mass m_0^2 at the Grand Unification scale
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- One interesting thing is that the gaugino masses evolve in the same way as the gauge couplings:
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  $4\pi dm_i^2/dt = -C_a^i 4M_a^2\alpha_a + |Y_{ijk}|^2[(m_i^2 + m_j^2 + m_k^2 + A_{ijk}^2)]/4\pi$
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- Weakly interacting particles tend to be lighter, particular those affected by large Yukawas.
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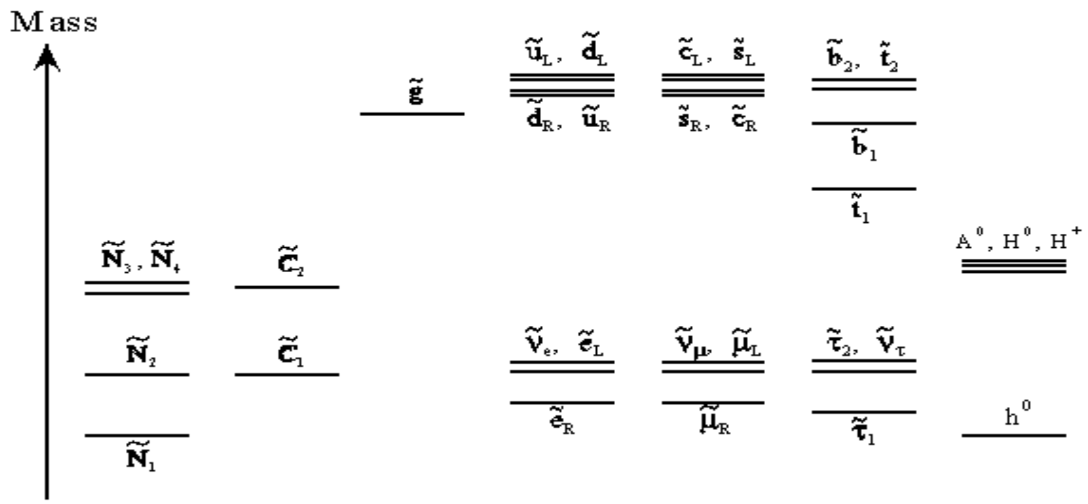

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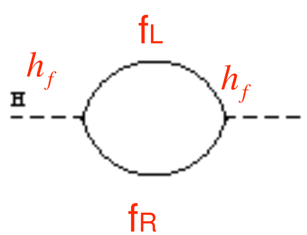

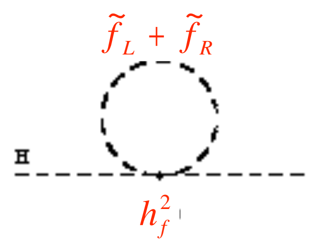

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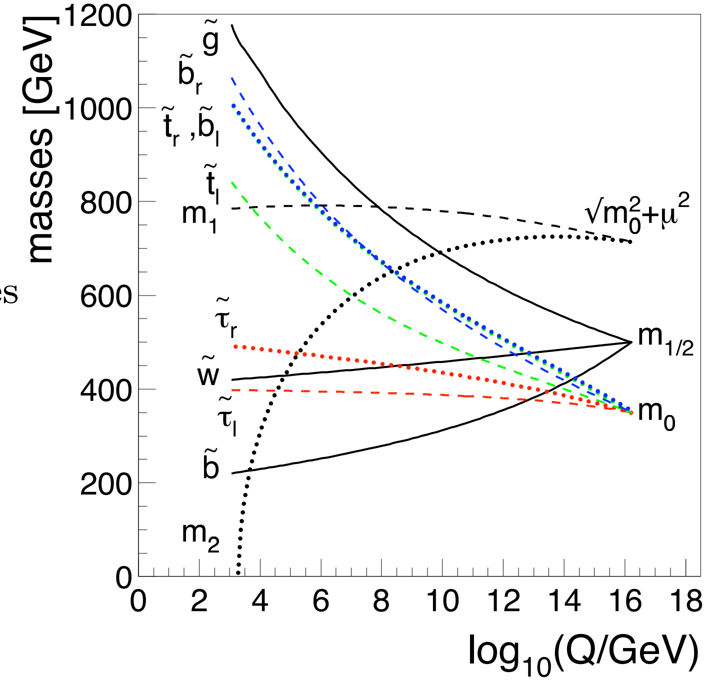
In the evolution of masses from high energy scales
$\longrightarrow$ a negative Higgs mass parameter is induced
via radiative corrections

$\Longrightarrow$ *important top quark effects!*



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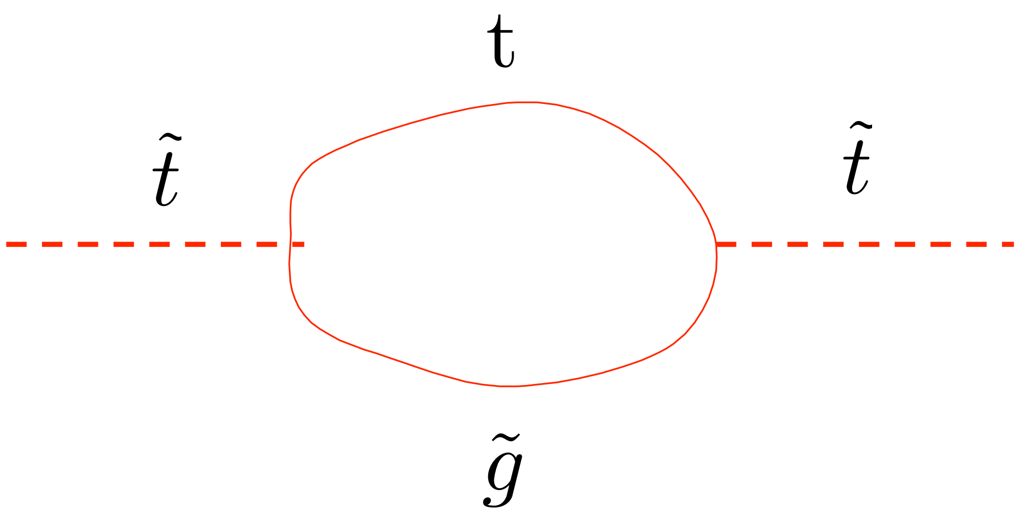
Gluino Contributions to the Stop Masses



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$$\frac{1}{\alpha_i(Q)} = \frac{b_i}{2\pi} \ln\left(\frac{Q}{M_Z}\right) + \frac{1}{\alpha_i(M_Z)} \tag{8}$$

- Unification of gauge couplings would occur if there is a given scale at which couplings converge.

$$\frac{1}{\alpha_3(M_Z)} = \frac{b_3 - b_2}{b_1 - b_2}\frac{1}{\alpha_1(M_Z)} + \frac{b_3 - b_1}{b_2 - b_1}\frac{1}{\alpha_2(M_Z)} \tag{9}$$

- This leads to a relation between $\alpha_3(M_Z)$ and $\sin^2\theta_W(M_Z) = \alpha_1^{SM}/\left(\alpha_1^{SM} + \alpha_2^{SM}\right)$.

Rules to compute the beta-functions

- The one-loop beta-functions for the $U(1)$ and $SU(N)$ gauge couplings are given by $(Q = T_3 + Y)$,

$$\begin{aligned} \frac{5}{3} b_1 &= -\frac{2}{3} \sum_f y_f^2 - \frac{1}{3} \sum_s y_s^2 \\ b_N &= \frac{11\ N}{3} - \frac{n_f}{3} - \frac{n_S}{6} - \frac{2N}{3} n_A \end{aligned} \tag{10}$$

- In the above, $y_{f,s}$ are the hypercharges of the charged chiral fermion and scalar fields, $n_{f,s}$ are the number of fermions and scalars in the fundamental representation of $SU(N)$, n_A are fermions in the adjoint representation and the factor 5/3 is just a normalization factor, so that over one generation

$$Tr\left[T^3 T^3\right] = \frac{3}{5} Tr\left[y_f^2\right] \tag{11}$$

- The coupling g_1 is not asymptotically free, but becomes strong at scales far above M_{Pl}, where the effective theory description breaks down anyway.
- A full generation contributes to the same amount to all β-functions. It does not affect the unification conditions. ($\beta_{\text{gen}}^{SM} = -4/3$; $\beta_{\text{gen}}^{\text{SUSY}} = -2$.)
- Only incomplete $SU(5)$ representations affect one-loop unification.

$$\begin{aligned} b_1^{SM} &= -41/10, \quad b_2^{SM} = 19/6, \quad b_3^{SM} = 7. \\ b_1^{\text{SUSY}} &= -33/5, \quad b_2^{\text{SUSY}} = -1, \quad b_3^{\text{SUSY}} = 3. \end{aligned} \tag{12}$$

$$\begin{aligned} M_G = M_Z \exp\left[\left(\frac{1}{\alpha_1(M_Z)} - \frac{1}{\alpha_2(M_Z)}\right)\frac{2\pi}{b_2 - b_1}\right] \\ \frac{1}{\alpha_3(M_Z)} = \frac{b_3 - b_1}{b_2 - b_1}\frac{1}{\alpha_2(M_Z)} + \frac{b_3 - b_2}{b_1 - b_2}\frac{1}{\alpha_1(M_Z)} \end{aligned} \tag{13}$$

Results

$b_1 = -6 - 3N_H/5, \qquad b_2 = -N_H, \qquad b_3 = 3$

$1/\alpha_1(M_Z) \simeq 60 \qquad 1/\alpha_2(M_Z) \simeq 30$

In the above, N_H is the number of pair of Higgs doublets. With these results we can compute the strong gauge coupling at M_Z

$$\frac{1}{\alpha_3(M_Z)} = \frac{15 - 7N_H}{25 + 3N_H} 30 \tag{1}$$

The result depends strongly on the number of Higgs doublets. For N_H equal to zero, we get $\alpha_3(M_Z) = 1/18 \simeq 0.05$. For $N_H = 1$, instead, we get $\alpha_3(M_Z) \simeq 0.120$!! Excellent prediction !

For $N_H = 2$ we get, again, a very bad result $\alpha_3(M_Z) \simeq 1$.

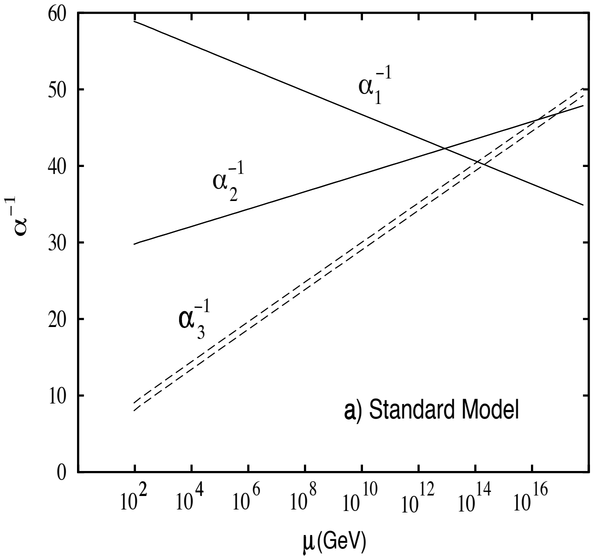
Finally, for $N_H = 1$,

$M_G = M_Z \exp(30 \times 2\pi \times (5/28)) \simeq 10^{16}\text{GeV}$

Grand Unification scale is very close to the Planck scale, suggesting a Unified description of particle interactions.

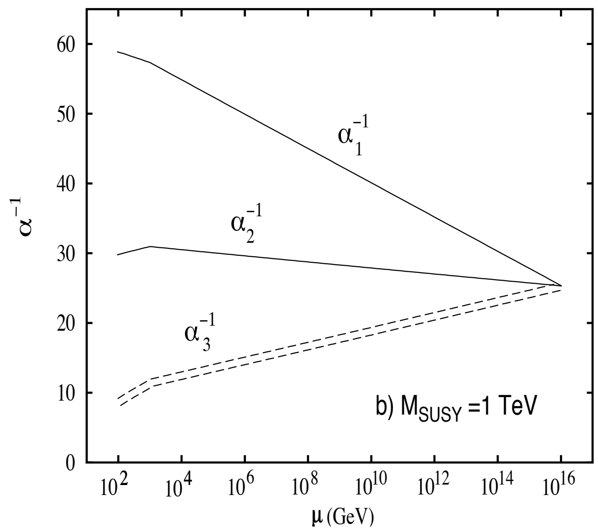
SM:

Couplings tend to converge at high energies, but unification is quantitatively ruled out.



MSSM:

Unification at $\alpha_{GUT} \simeq 0.04$ and $M_{GUT} \simeq 10^{16}$ GeV.



Experimentally, $\alpha_3(M_Z) \simeq 0.118 \pm 0.004$ Bardeen, Carena, Pokorski & C.W.

in the MSSM: $\alpha_3(M_Z) = 0.127 - 4(\sin^2\theta_W - 0.2315) \pm 0.008$

Remarkable agreement between Theory and Experiment!!

R-Parity

- A solution to the proton decay problem is to introduce a discrete symmetry, called R-Parity. In the language of component fields,

$$R_P = (-1)^{3B+2S+L} \tag{7}$$

- All Standard Model particles have $R_P = 1$.
- All supersymmetric partners have $R_P = -1$.
- All interactions with odd number of supersymmetric particles, like the Yukawa couplings induced by $P[\Phi]_{\rm new}$ are forbidden.
- Supersymmetric particles should be produced in pairs.
- The lightest supersymmetric particle is stable.
- Good dark matter candidate. Missing energy at colliders.

SUSY Collider Phenomenology


Chicago
Booster
CDF
DØ
Tevatron
p̄ source
Main Injector
(new)

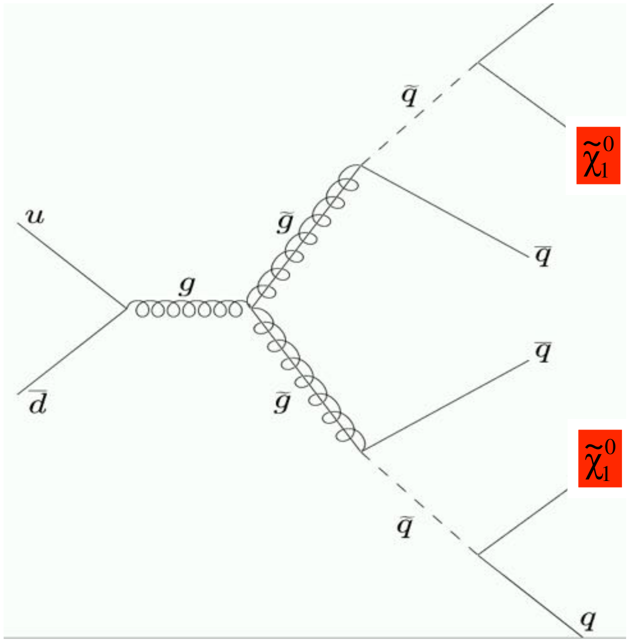
Preservation of R-Parity:
Supersymmetry at colliders

Gluino production and decay: Missing Energy Signature

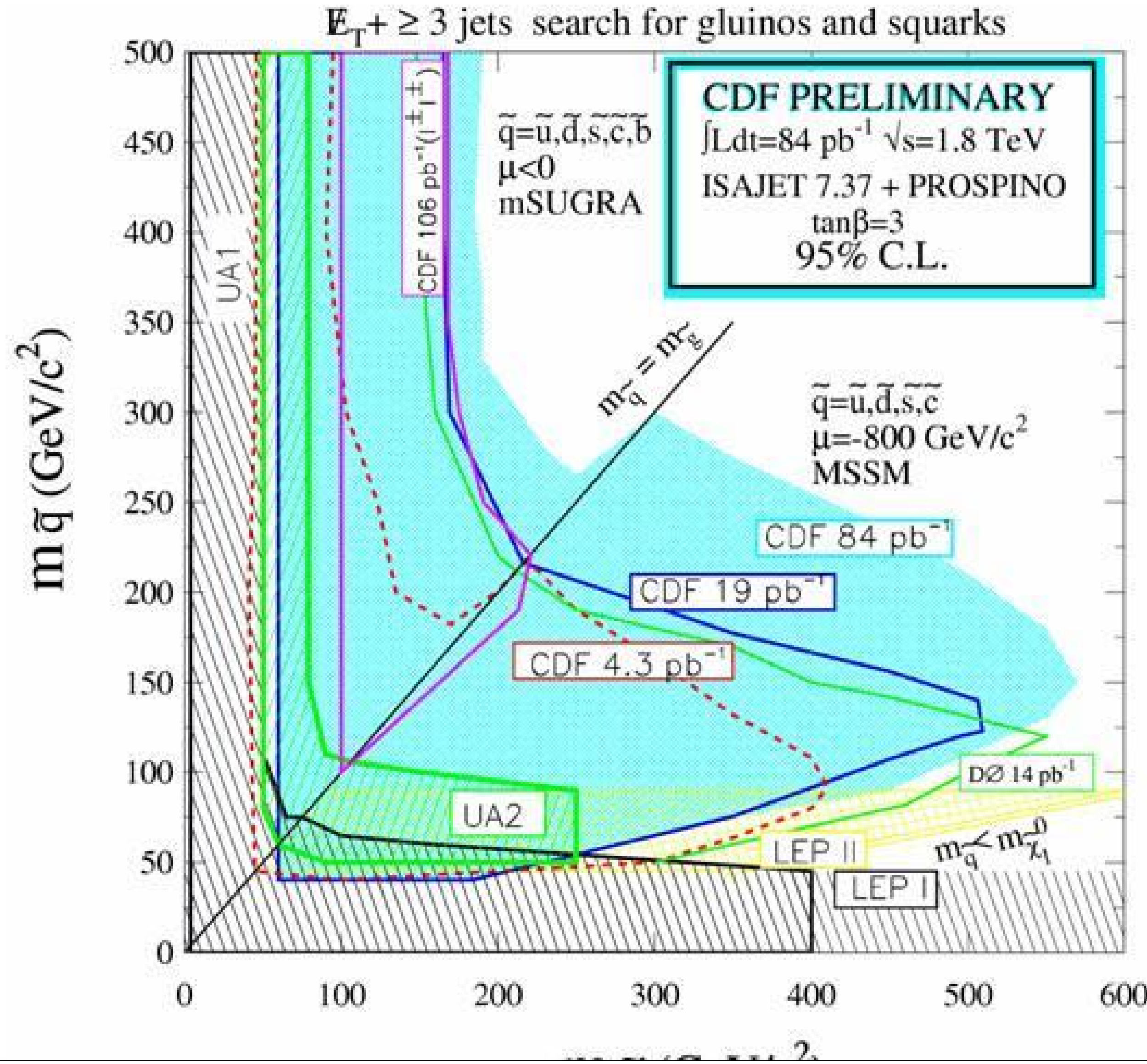
Supersymmetric Particles tend to be heavier if they carry color charges.

Particles with large Yukawas tend to be lighter.

Charge-less particles tend to be the lightest ones.



➢ **Lightest supersymmetric particle = Excellent Cold dark matter candidate.**

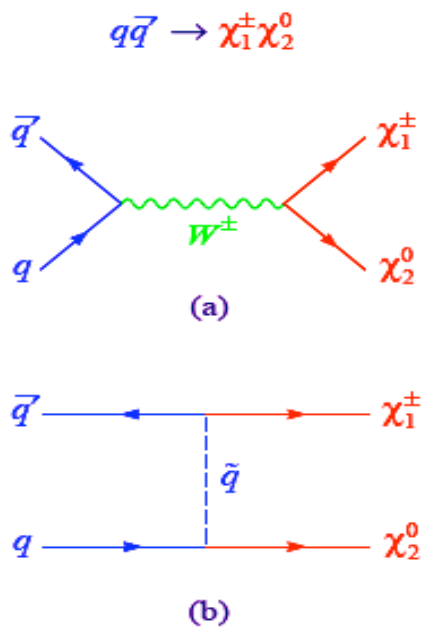


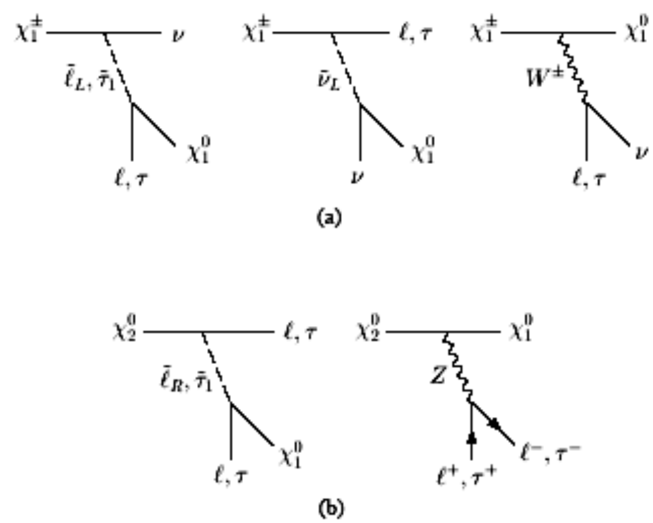
$\not{E}_T + \geq 3$ jets search for gluinos and squarks
CDF PRELIMINARY
$\int Ldt = 84\ pb^{-1}$ $\sqrt{s} = 1.8$ TeV
ISAJET 7.37 + PROSPINO
$\tan\beta = 3$
95% C.L.
$\tilde{q} = \tilde{u}, \tilde{d}, \tilde{s}, \tilde{c}, \tilde{b}$
$\mu < 0$
mSUGRA
$\tilde{q} = \tilde{u}, \tilde{d}, \tilde{s}, \tilde{c}$
$\mu = -800\ GeV/c^2$
MSSM
CDF 106 pb^{-1} ($l^{\pm} l^{\pm}$)
UA1
UA2
$m_{\tilde{q}} = m_{\tilde{g}}$
CDF 84 pb^{-1}
CDF 19 pb^{-1}
CDF 4.3 pb^{-1}
DØ 14 pb^{-1}
LEP II
LEP I
$m_{\tilde{q}} < m_{\tilde{\chi}_1^0}$
$m_{\tilde{q}}$ (GeV/c^2)

Trilepton Signatures at the Tevatron

Trileptons are associated production of chaginos and neutralinos, with subsequent decays into leptons. The final signal is three lepons and plenty of missing energy (neutralinos and neutrinos)

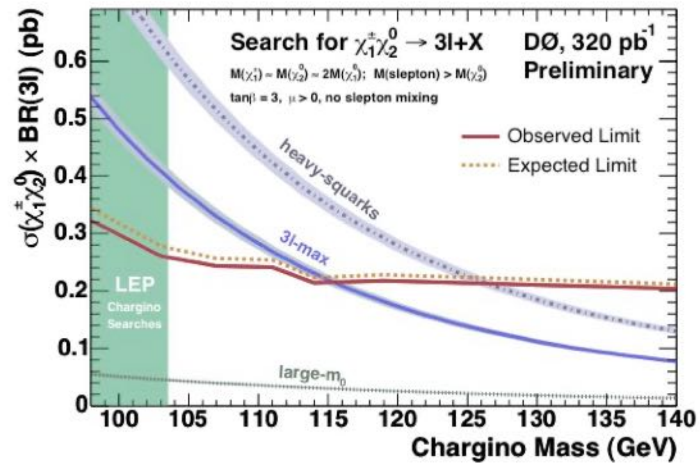
Main background: W and Z production.

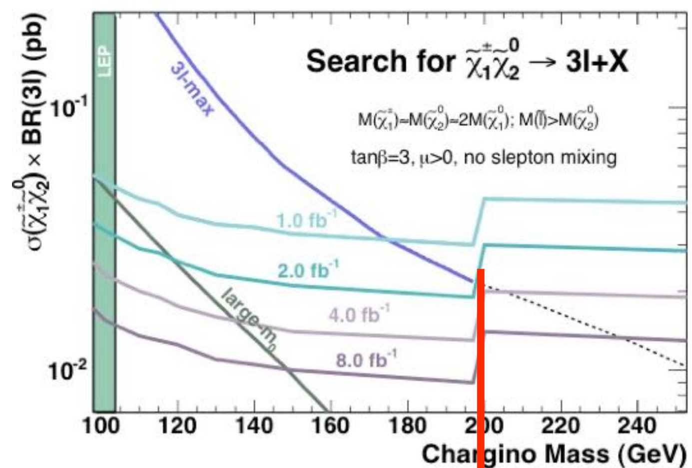




Trileptons: Present and Future

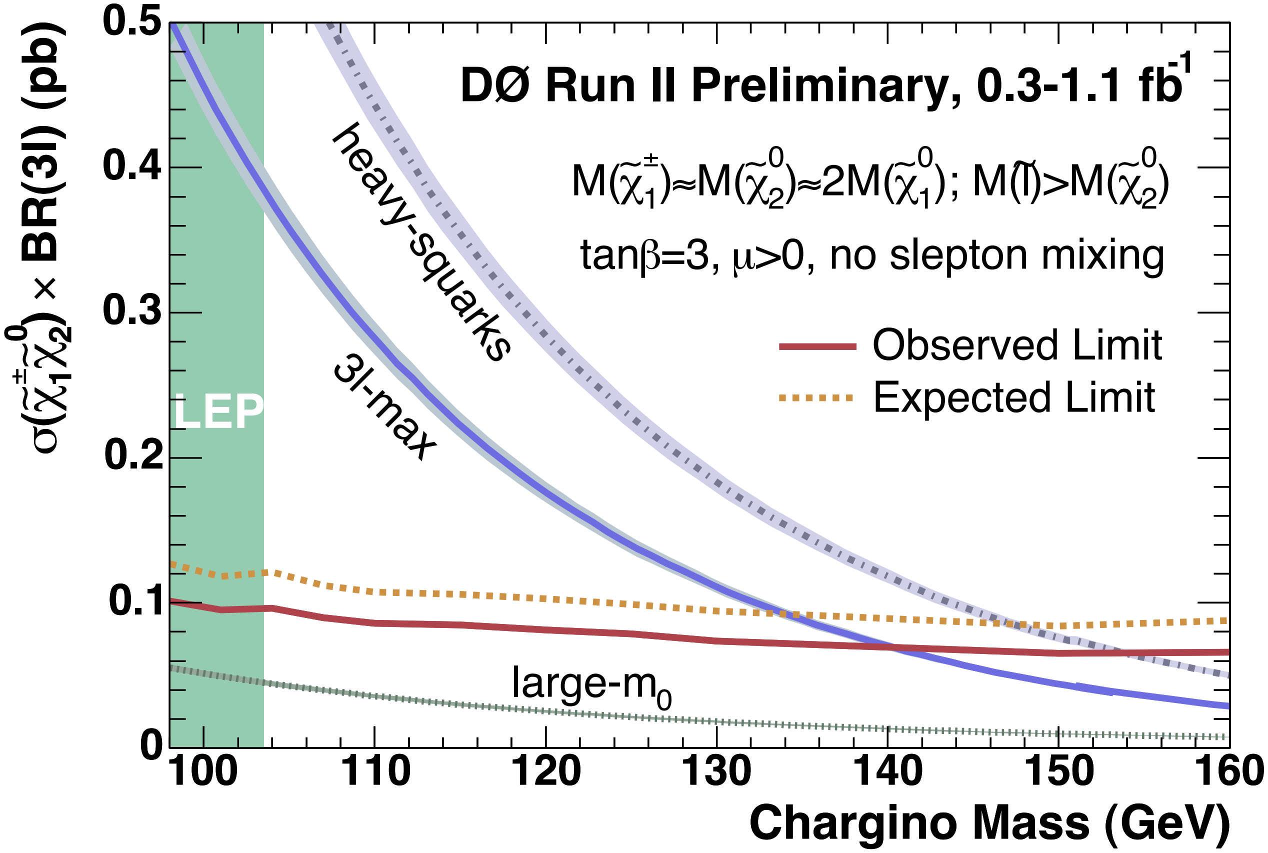
- **Now**: σx**BR<0.2–0.3 pb**
  - **3l–max scenario**:
    - **Sleptons light**
    - **Optimistic mSUGRA**
  - **Large m_0 scenario:**
    - **Sleptons heavy**
    - **Pessimistic mSUGRA**
  - **Current data probe optimistic scenario**
- **Future:**
  - **Cross section limit 0.05–0.01 pb**
  - **L=1 fb^{-1}: probe chargino masses up to 100–170 GeV/c^2**
  - **L=8 fb^{-1}: probe chargino masses up to 150–240 GeV/c^2**

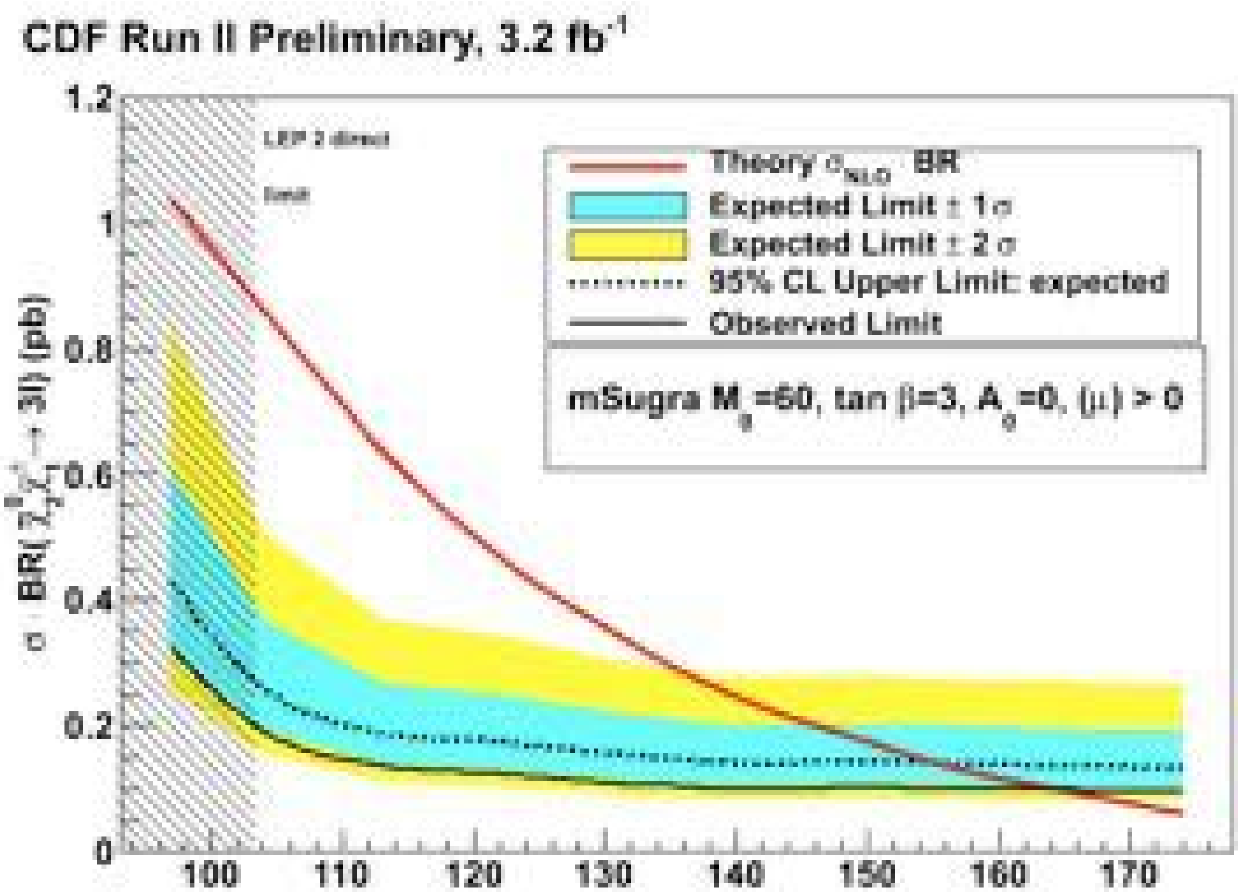




Preferred by precision data

No signal observed. Exclusion limits

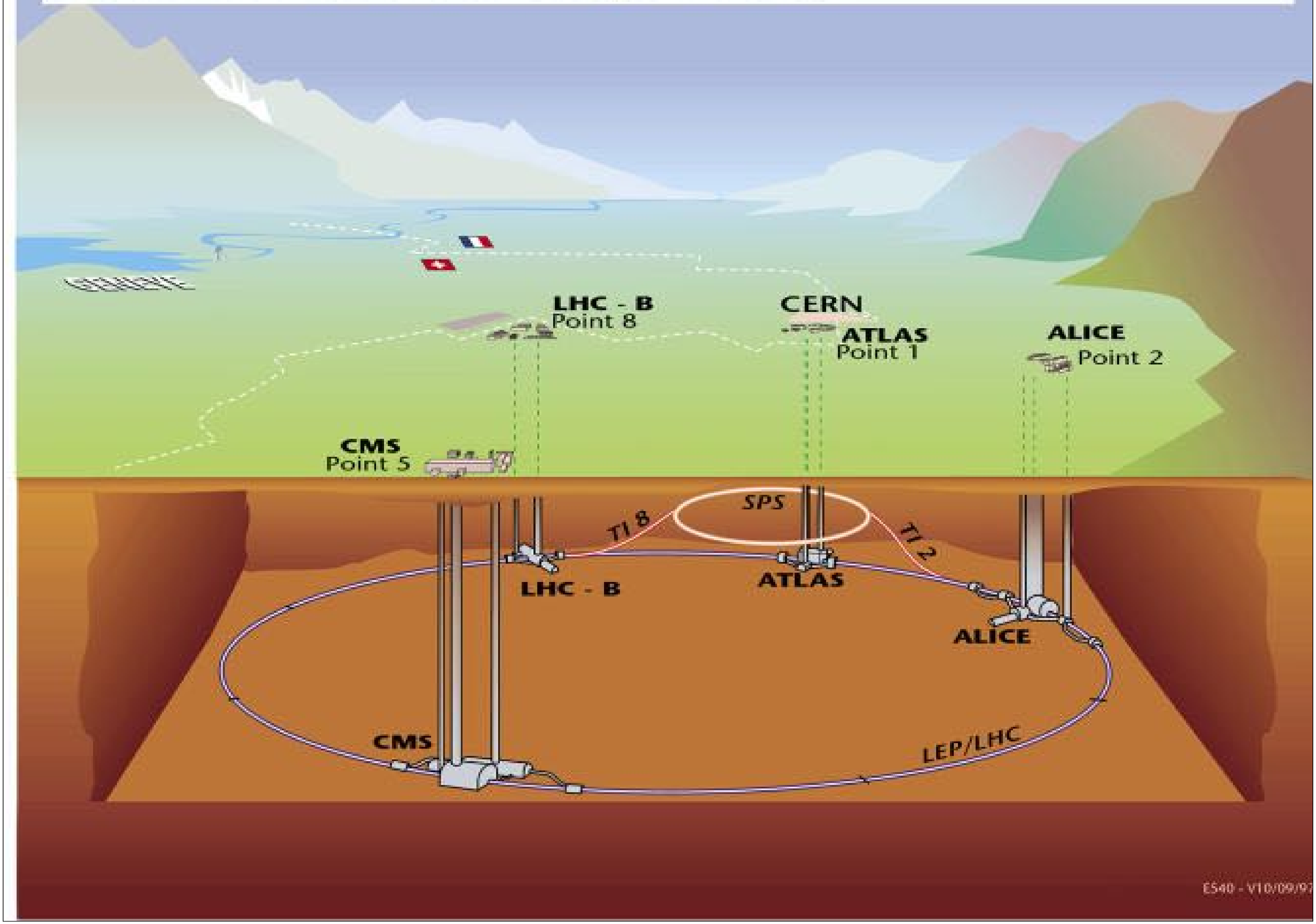



CDF Run II Preliminary, 3.2 fb^{-1}
LEP 2 direct
limit
Theory σ_{NLO} BR
Expected Limit ± 1σ
Expected Limit ± 2 σ
95% CL Upper Limit: expected
Observed Limit
mSugra M_0=60, tan β=3, A_0=0, (μ) > 0
σ · BR($\tilde{\chi}^0_2\tilde{\chi}^{\pm}_1 \to 3l$) (pb)
0
0.2
0.4
0.6
0.8
1
1.2
100
110
120
130
140
150
160
170

Beyond the Tevatron

- Searches at the Tevatron are limited by phase space.
- Tevatron is a proton-antiproton collider with a center of mass energy of 2 TeV. Partons c.m.e. is smaller, and strongly interacting (weakly interacting) sparticles with masses beyond 500 (250) GeV may not be discovered.
- Tevatron has still a chance, and we all hope that something exciting will happen there in the near future.
- Searches for new physics will continue in 2008 at the LHC, a proton proton collider with c.m.e. of about 14 TeV !
- Strongly interacting (colored) particles, with masses up to 3--4 TeV may be explored !

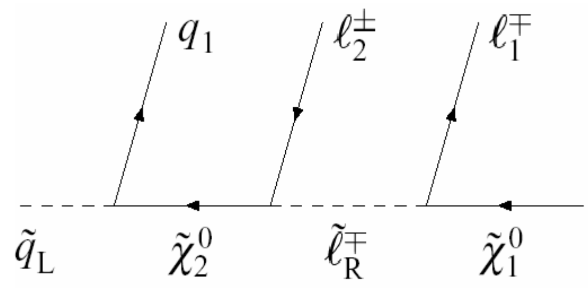
Overall view of the LHC experiments.



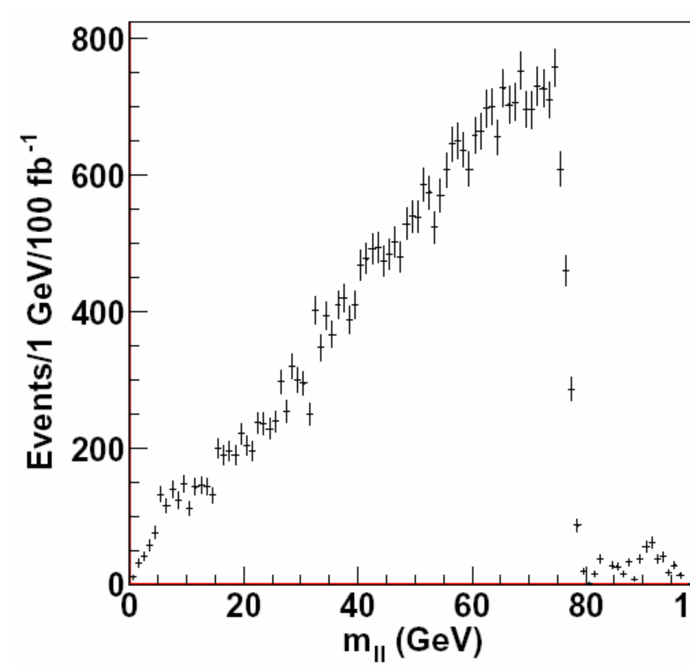
Searches at the LHC

New particle searches at the LHC are induced by the cascade decay of strongly interacting particles.

By studying the kinematic distributions of the decay products one can determine the masses of produced particles, including the LSP.



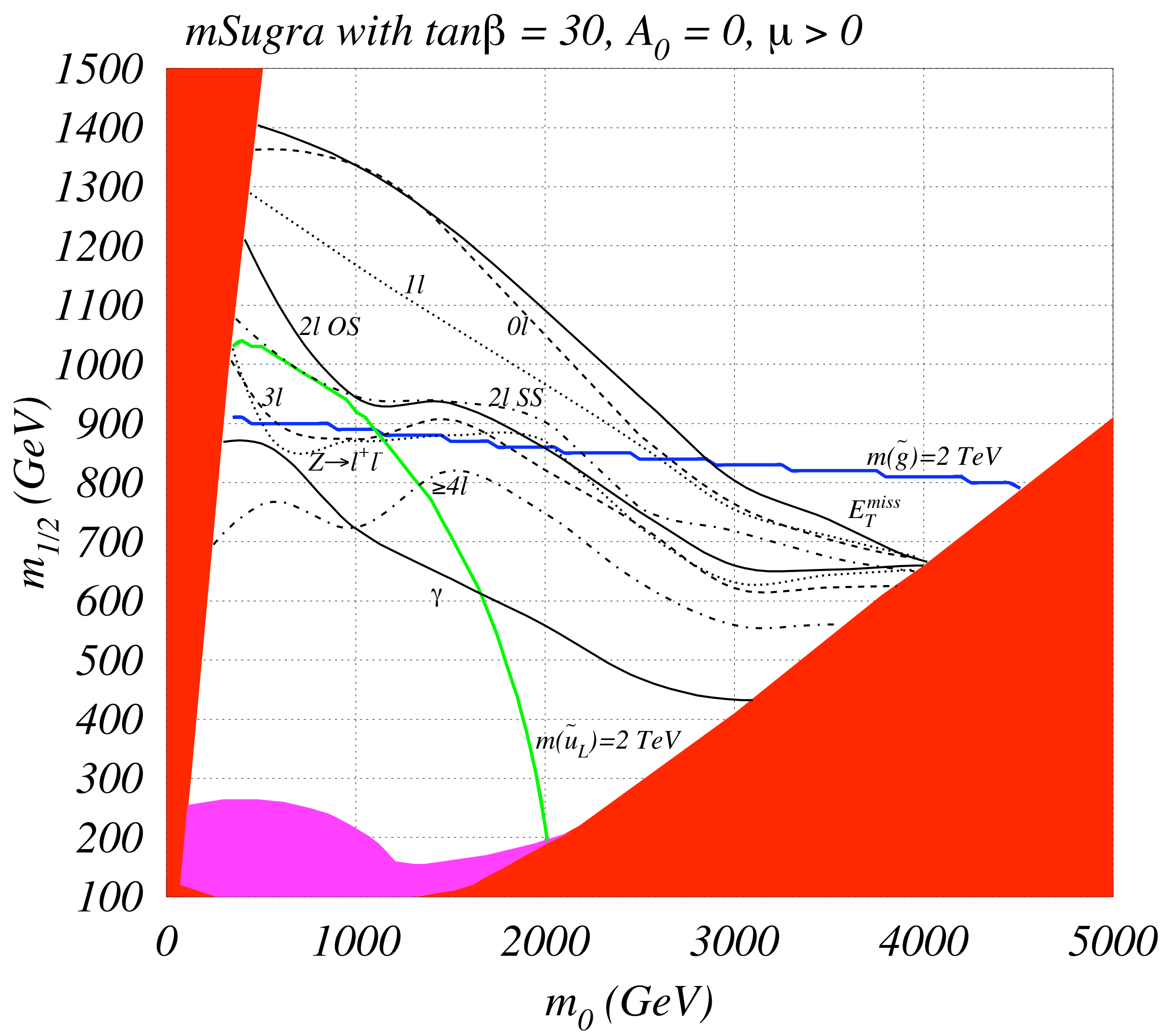
$$
\begin{aligned}
(m^2_{ll})^{\rm edge} &= \frac{\left(m^2_{\tilde{\chi}^0_2} - m^2_{\tilde{l}_R}\right)\left(m^2_{\tilde{l}_R} - m^2_{\tilde{\chi}^0_1}\right)}{m^2_{\tilde{l}_R}} \\
(m^2_{qll})^{\rm edge} &= \frac{\left(m^2_{\tilde{q}_L} - m^2_{\tilde{\chi}^0_2}\right)\left(m^2_{\tilde{\chi}^0_2} - m^2_{\tilde{\chi}^0_1}\right)}{m^2_{\tilde{\chi}^0_2}} \\
(m^2_{ql})^{\rm edge}_{\rm min} &= \frac{\left(m^2_{\tilde{q}_L} - m^2_{\tilde{\chi}^0_2}\right)\left(m^2_{\tilde{\chi}^0_2} - m^2_{\tilde{l}_R}\right)}{m^2_{\tilde{\chi}^0_2}} \\
(m^2_{ql})^{\rm edge}_{\rm max} &= \frac{\left(m^2_{\tilde{q}_L} - m^2_{\tilde{\chi}^0_2}\right)\left(m^2_{\tilde{l}_R} - m^2_{\tilde{\chi}^0_1}\right)}{m^2_{\tilde{l}_R}} \\
(m^2_{qll})^{\rm thres} &= [(m^2_{\tilde{q}_L} + m^2_{\tilde{\chi}^0_2})(m^2_{\tilde{\chi}^0_2} - m^2_{\tilde{l}_R})(m^2_{\tilde{l}_R} - m^2_{\tilde{\chi}^0_1}) \\
&\quad -(m^2_{\tilde{q}_L} - m^2_{\tilde{\chi}^0_2})\sqrt{(m^2_{\tilde{\chi}^0_2} + m^2_{\tilde{l}_R})^2(m^2_{\tilde{l}_R} + m^2_{\tilde{\chi}^0_1})^2 - 16 m^2_{\tilde{\chi}^0_2} m^4_{\tilde{l}_R} m^2_{\tilde{\chi}^0_1}} \\
&\quad +2m^2_{\tilde{l}_R}(m^2_{\tilde{q}_L} - m^2_{\tilde{\chi}^0_2})(m^2_{\tilde{\chi}^0_2} - m^2_{\tilde{\chi}^0_1})]/(4m^2_{\tilde{l}_R} m^2_{\tilde{\chi}^0_2})
\end{aligned}
$$



Weiglein et al. (2004)

How well can the LHC do ?

Baer, Balazs, Belyaev, Kropovnickas and Tata '03.



One stop could be light

- What happens if all squarks and gluinos are heavy, except for a light stop ?
- Charginos and neutralinos are still around, providing the proper dark matter annihilation cross section.
- Both stops cannot be light in the MSSM. Otherwise, the physical Higgs mass would be too light.
- Let’s then study the case of a single light stop. This case is motivated by the question of electroweak baryogenesis.

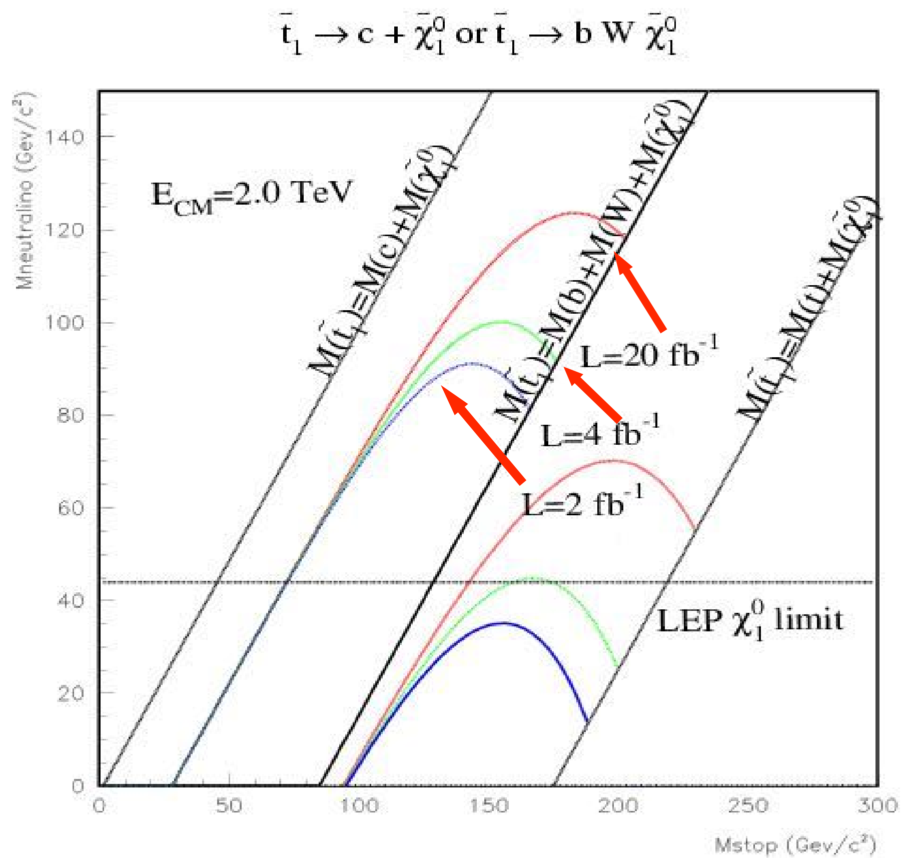
Tevatron Stop Reach when two body decay channel is dominant

Main signature:

2 or more jets plus missing energy

2 or more Jets with $E_T > 15$ GeV

Missing $E_T > 35$ GeV



Demina, Lykken, Matchev,Nomerotsky '99

Stop-Neutralino Mass Difference: Information from the Cosmos

M. Carena, C. Balazs, C.W., PRD70:015007, 2004

M. Carena, C. Balazs, A. Menon, D. Morrissey, C.W., Phys. Rev. D71:075002, 2005.

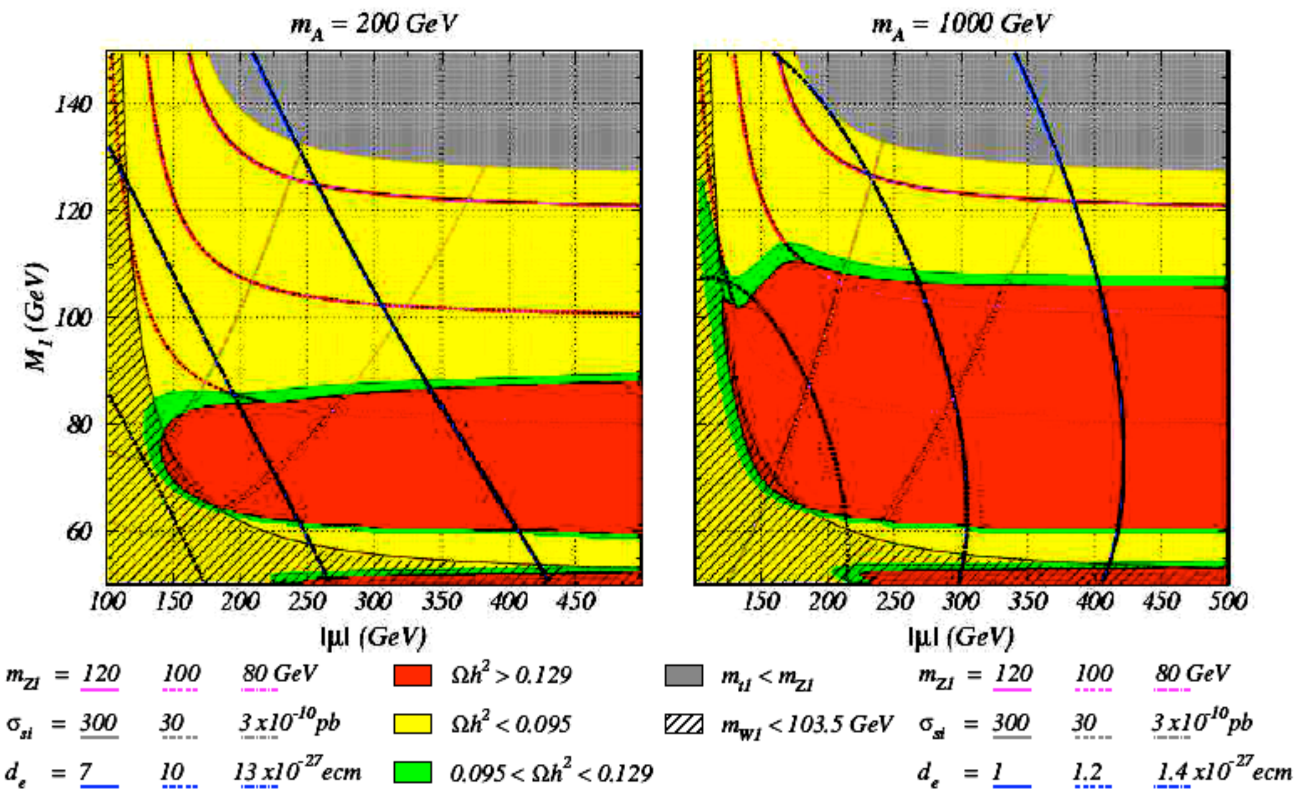
- If the neutralino provides the observed dark matter relic density, then it must be stable and lighter than the light stop.

- Relic density is inversely proportional to the neutralino annihilation cross section.

If only stops, charginos and neutralinos are light, there are three main annihilation channels:

1. Coannihilation of neutralino with light stop or charginos: Small mass differences.
2. s-channel annihilation via Z or light CP-even Higgs boson
3. s-channel annihilation via heavy CP-even Higgs boson and CP-odd Higgs boson

Light Stop and Relic Density Constraints

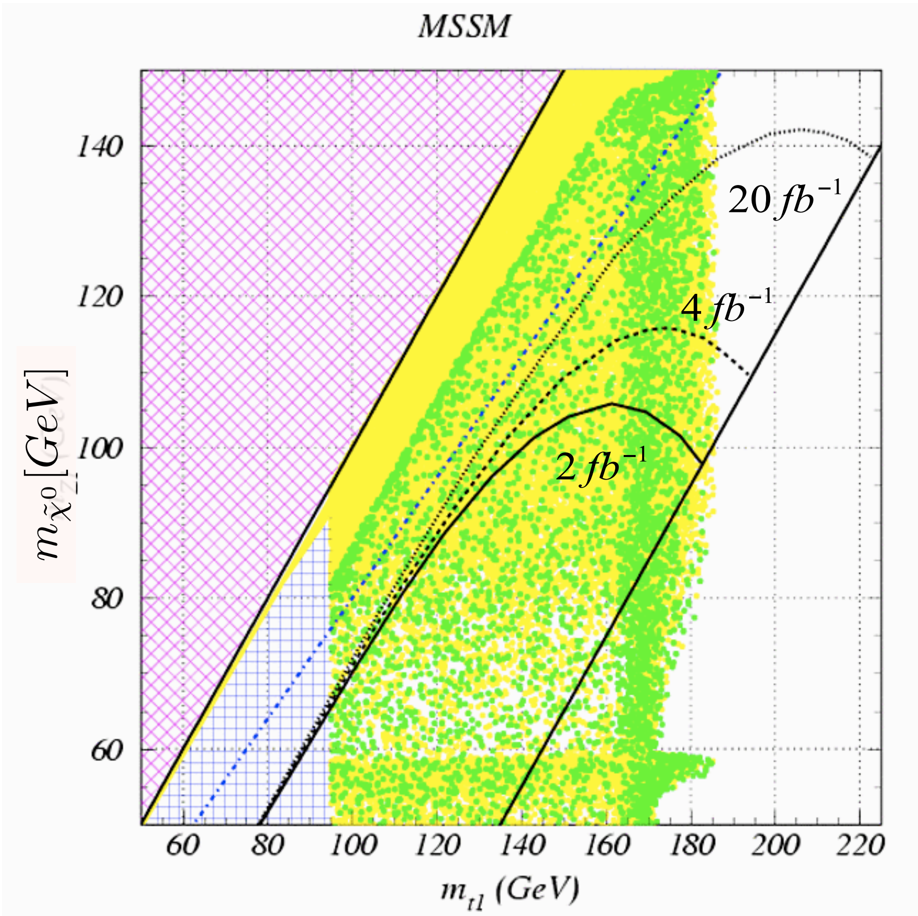


$\tan\beta = 7$

Tevatron stop searches and dark matter constraints

$\tilde{t}_1 \rightarrow c\, \tilde{\chi}_1^0$

Carena, Balazs and C.W. '04



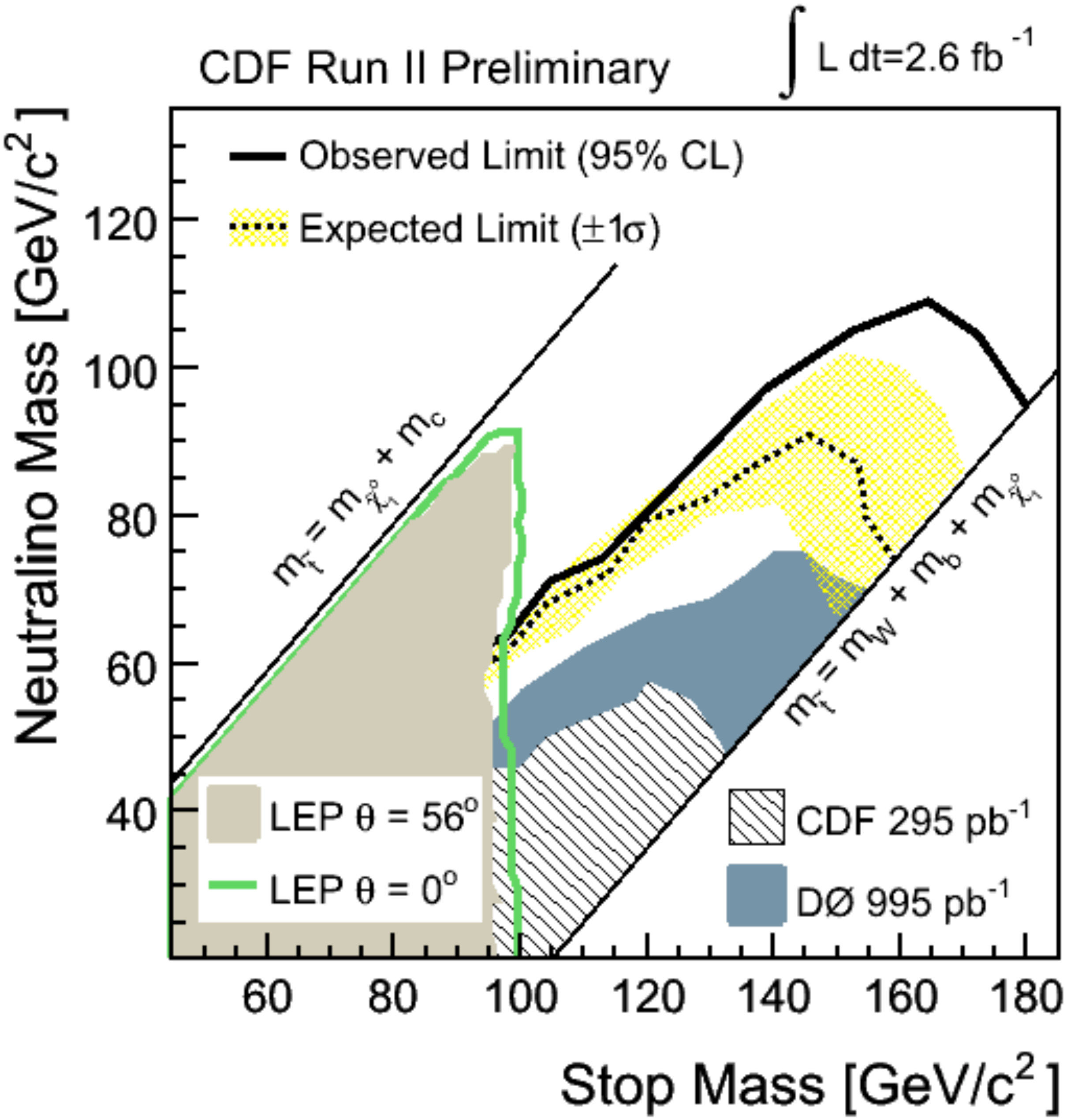
Green: Relic density consistent with WMAP measurements.

Searches for light stops difficult in stop-neutralino coannihilarion region.

LHC will have equal difficulties.

But, LHC can search for stops from gluino decays into stops and tops.
Stops may be discovered for gluino masses lower than 900 GeV, even if the stop-neutralino mass difference is as low as 10 GeV !

Kraml, Raklev '06, Martin 08



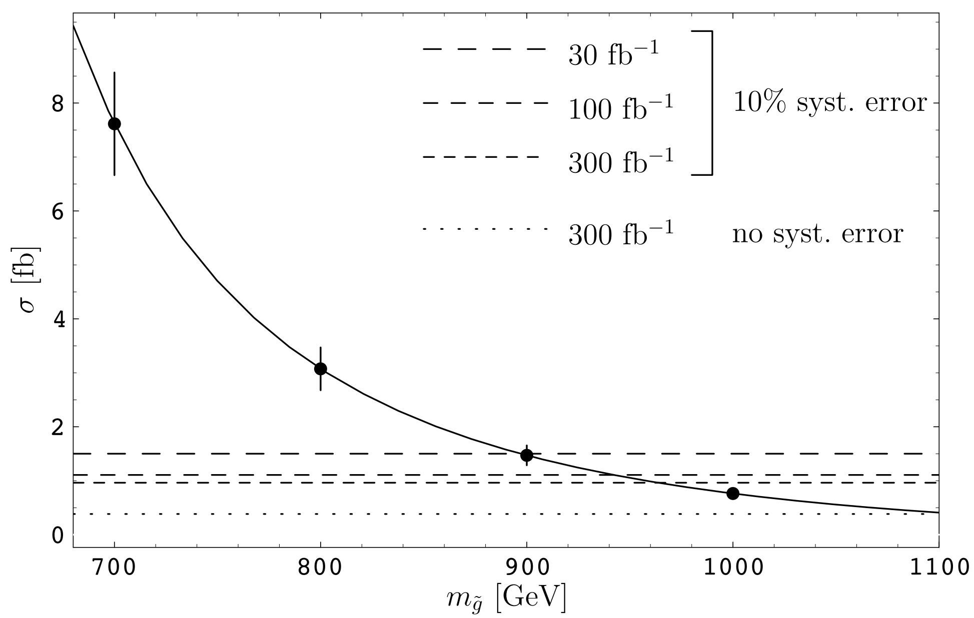
CDF Run II Preliminary
∫ L dt=2.6 fb^-1
Observed Limit (95% CL)
Expected Limit (±1σ)
LEP θ = 56°
LEP θ = 0°
CDF 295 pb^-1
DØ 995 pb^-1
Neutralino Mass [GeV/c^2]
Stop Mass [GeV/c^2]

Stops from Gluino Decays

Kraml, Raklev '06, Martin 08

Take advantage of Majorana character of gluino: $\tilde{g} \to \tilde{t}_1 \bar{t},\ \tilde{t}_1^* t.$ Production of equal sign tops

- Two same-sign leptons with $p_T > 20$ GeV.
- Two b-tagged jets with $p_T > 50$ GeV. (b-tag eff. 43%)
- $\not{E}_T > 100$ GeV. Invariant mass $m_{bl} < 160$ GeV



Efficient stop search channel up to gluino masses of about 1 TeV

Carena, Freitas, C.W.'08

Alternative Channels at the LHC

- When the stops and neutralino mass difference is small, the jets will be soft.
- One can look for the production of stops in association with jets or photons. Signature: Jets or photons plus missing energy

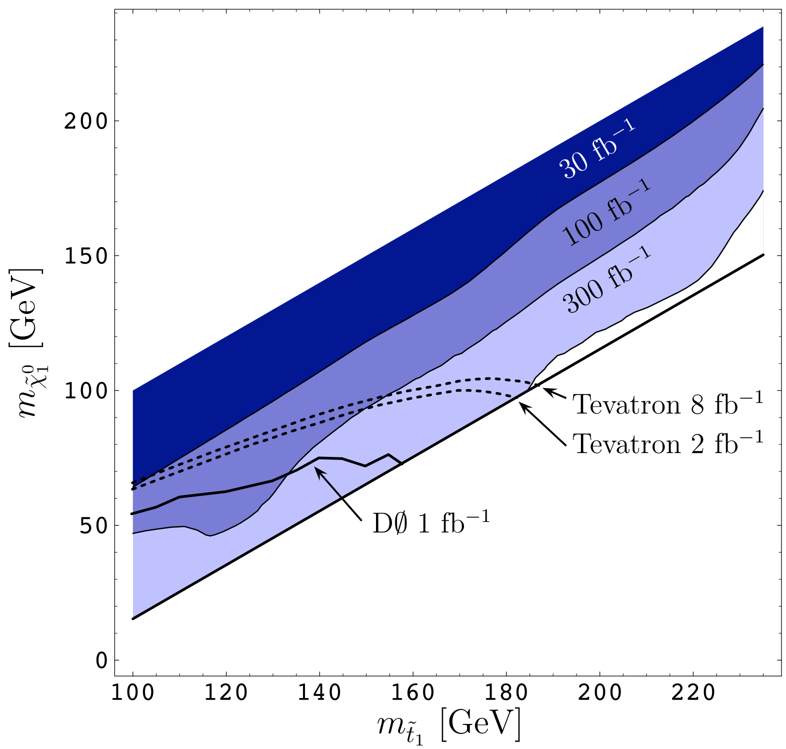
M. Carena, A. Freitas, C.W., arXiv:0808.2298

- Photon plus missing energy searches have the advantage of being cleaner, but they suffer from low statistics and large systematics
- Jet plus missing energy searches have larger backgrounds but have the advantage of having much larger production cross section compared to the photon case
- Hard photons and jets recoiling against missing energy have been simulated at the LHC experiments in the search for large extra dimensions, and we will make use of the backgrounds computed for that purpose.

Jets plus missing Energy

M. Carena, A. Freitas, C.W.'08

1. Require one hard jet with $p_T > 100$ GeV and $|\eta| < 3.2$ for the trigger.
2. Large missing energy $\not{E}_T > 1000$ GeV.



Including systematics associated with jet and missing energy determination. Dominant missing energy background, coming from Z's, calibrated with the electron channel.

Excellent reach until masses of the order of 220 GeV and larger.

Full region consistent with EWBG will be probed by combining the LHC with the Tevatron searches.

Alternative SUSY Breaking scenario: Gauge Mediation

Supersymmetry Breaking is transmitted via gauge interactions

Particle Masses depend on the strength of their gauge interactions.

Spectrum of supersymmetric particles very similar to the case of the Minimal Supergravity Model for large values of $M_{1/2}$:

Sparticle Masses

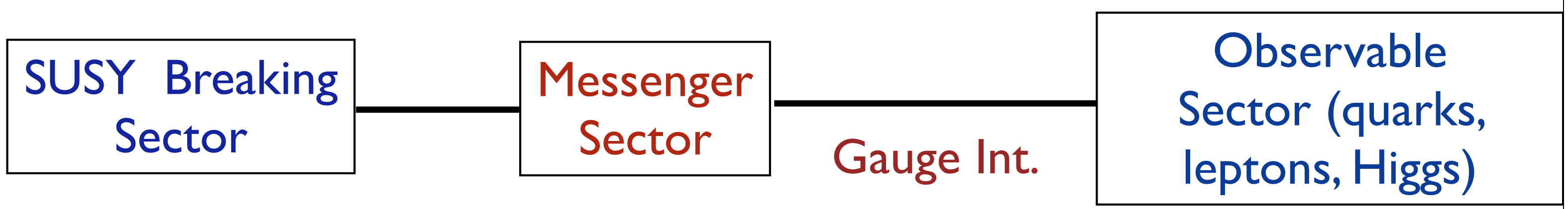
$\frac{M_i}{M_j} = \frac{\alpha_i}{\alpha_j}$

$\frac{m_{\tilde{q}}}{m_{\tilde{l}}} \simeq \frac{\alpha_3}{\alpha_i}$ $\quad\quad\quad\quad$ $(m_{\tilde{f}} \simeq \frac{\alpha}{4\pi}\frac{F}{M})$

Lightest SM–Sparticle tends to be a Bino or a Higgsino

Gauge Mediated SUSY Breaking

- Supersymmetry breaking is transmitted to the observable sector via (flavor blind) gauge interactions



- Messenger sector in complete representations of SU(5) and vector-like.

- Minimal model: One $(5,\bar{5}) \equiv (3,2) + (\bar{3},\bar{2})$

$$W = \lambda\, S\, 3\, \bar{3} + \gamma\, S\, 2\, \bar{2}, \qquad < S >= S + F_S \theta^2,$$

with S a singlet field parametrizing SUSY breaking and the messenger mass

Spectrum of Sparticles (more details later)

- Gaugino masses fulfill the standard unification relations,

$$M_i \propto \frac{\alpha_i}{4\pi}\frac{F_S}{S}, \qquad\qquad \frac{M_i}{M_j} = \frac{\alpha_i}{\alpha_j}$$

- Scalar masses at the messenger scale are also governed by their color structure. For instance,

$$m_{\tilde{q},H} \propto \frac{\alpha_{3,2}}{4\pi}\frac{F_S}{S},$$

- This implies that, independently of the messenger scale, there are large negative corrections to the Higgs mass parameter, triggering EWSB
- The requirement of a weak scale spectrum demands

$$\Lambda \equiv \frac{F_S}{S} = \mathcal{O}(10^5 \text{ GeV})$$

- The scale of SUSY breaking has important consequences, for instance it determines the gravitino mass and interactions (and therefore the nature of the LSP). Lightest superpartner tends to be a Bino.

Gravitino

- When standard symmetries are broken spontaneously, a massless boson appears for every broken generator.
- If the symmetry is local, this bosons are absorbed into the longitudinal components of the gauge bosons, which become massive.
- The same is true in supersymmetry. But now, a massless fermion appears, called the Goldstino.
- In the case of local supersymmetry, this Goldstino is absorbed into the Gravitino, which acquires mass $m_{\tilde{G}} = F/M_{Pl}$, with F the order parameter of SUSY breaking.
- The coupling of the Goldstino (gravitino) to matter is proportional to $1/F = 1/(m_{\tilde{G}} M_{\rm Pl})$, and couples particles with their superpartners.
- Masses of supersymmetric particles is of order F/M, where M is the scale at which SUSY is transmitted.

Decay Width of NLSP into Gravitino

- In low energy supersymmetry breaking models, the SUSY breaking scale $10\text{TeV} \leq \sqrt{F} \leq 10^3$ TeV.
- The gravitino mass is very small in this case $10^{-1}\text{eV} \leq m_{\tilde{G}} \leq 10^3$ eV.
- Since the gravitino is the lightest supersymmetric particle, then the lightest SM superpartner will decay into it.
- It is easy to extract the decay width on dimensional grounds
- Just assume that the lightest SM partner is a photino, for instance, and it decays into an almost massless gravitino and a photon. Then,

$$\Gamma(\tilde{\gamma} \to \gamma \tilde{G}) \simeq \frac{m_{\tilde{\gamma}}^5}{16\pi F^2} \simeq \frac{m_{\tilde{\gamma}}^5}{16\pi m_{\tilde{G}}^2 M_{Pl}^2} \tag{2}$$

where we have used the fact that $[F] = 2$.

Gauge-Mediated, Low-energy SUSY Breaking Scenarios

- Special feature $\longrightarrow$ LSP: light (gravitino) Goldstino:

$$m_{\tilde{G}} \sim \frac{F}{M_{Pl}} \simeq 10^{-6} - 10^{-9}\text{GeV}$$

If R-parity conserved, heavy particles cascade to lighter ones and

NLSP $\longrightarrow$ SM partner + $\tilde{G}$

- Signatures: The NLSP (Standard SUSY particle) decays

decay length $\quad L \sim 10^{-2}\text{cm} \left(\frac{m_{\tilde{G}}}{10^{-9}\text{GeV}}\right)^2 \times \left(\frac{100\text{GeV}}{M_{\text{NLSP}}}\right)^5$

⋆ NLSP can have prompt decays:

Signature of SUSY pair: 2 hard photons, (H's, Z's) + $\not{E}_T$ from $\tilde{G}$

⋆ macroscopic decay length but within the detector:

displaced photons; high ionizing track with a kink to a minimum ionizing track

(smoking gun of low energy SUSY)

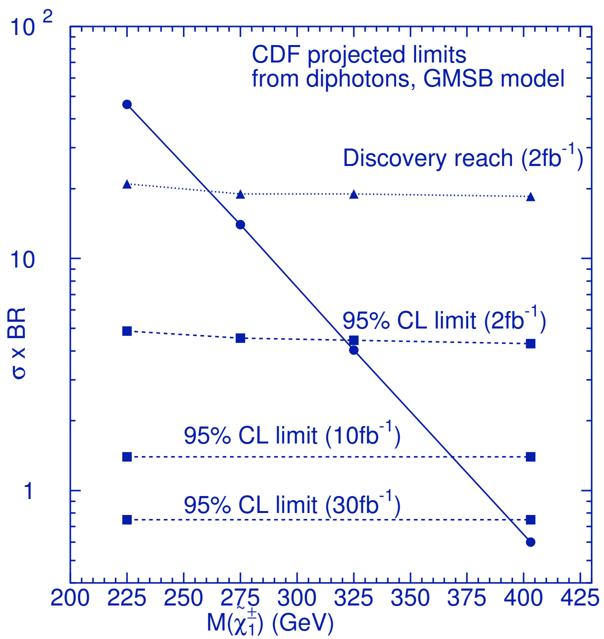
⋆ decay well outside the detector: $\not{E}_T$ like SUGRA

Gauge-Mediated Tevatron Reach

- Bino-like NLSP: $\tilde{\chi}_1^0 \rightarrow \gamma \tilde{G}$

  Signal: $\gamma\gamma X \not{E}_T$

  $X = \ell$'s and/or jets



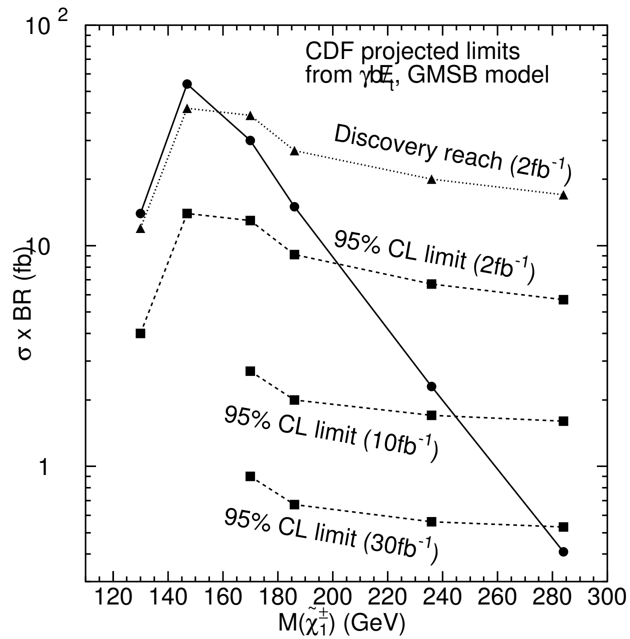
$M_{\tilde{\chi}^{\pm}} \sim 325$ GeV *(exclusion)* &

~ 260 GeV *(discovery)*

- Higgsino-like NLSP: $\tilde{\chi}_1^0 \rightarrow (h, Z, \gamma)\tilde{G}$

  Signal: γ b $\not{E}_T X$

  diboson signatures $(Z \rightarrow \ell\ell/\mathrm{jj};\ h \rightarrow b\bar{b})\not{E}_T$



$M_{\tilde{\chi}_1^{\pm}}$ sensitivity ~ 200 GeV for 2 fb^{-1}

Conclusions

- Supersymmetry is a symmetry that relates boson to fermion degrees of freedom. It provides the basis for an extension of the SM description of particle interactions.
- Fundamental property: No new couplings. Masses of supersymmetric particles depend on supersymmetry breaking scheme.
- If R-Parity is imposed and the gravitino is heavier than the lightest SM partner, then the lightest supersymmetric particle is a good dark matter candidate.
- Electroweak symmetry breaking is induced radiatively in a natural way, provided sparticle masses are of order 1 TeV. Unification of couplings is achieved.
- Signature at colliders: Missing Energy, provided by the LSP.